Valuing Employee Stock Options
Using a Lattice Model

By Les Barenbaum, Walt Schubert, and Bonnie O’Rourke

A recent FASB exposure draft on stock-option expensing would require the valuation of equity-based compensation awards at their grant date. Option value and the resulting expense are based upon models that capture the characteristics determining the value of a particular grant of employee options. The exposure draft discusses lattice valuation models that accommodate the often complex attributes of option plans that can change over time. The lattice model can explicitly capture expected changes in dividends and stock volatility over the expected life of the options, in contrast to the Black-Scholes option-pricing model, which uses weighted average assumptions about option characteristics. The authors’ objective is to provide an overview of how lattice models work and to provide insights into how lattice models can help ascertain the costs and benefits of various option-granting strategies.

Overview

A lattice structure, such as the binomial model, incorporates assumptions about employee exercise behavior over the life of each option grant and changes in expected stock-price volatility. This results in more-accurate option values and compensation expense.

Employee stock options have distinct characteristics. For example, it is typical for a large percentage of employees to exercise their options upon vesting. Other employees hold their options and exercise based upon their assessment of the expected future movement of the stock price. Lattice models provide a framework to capture the impact of these varying exercise patterns into the calculation of the value of the option. This impact can be material.

Another advantage of the lattice structure is the ability to incorporate expected changes in volatility over the life of the option. This is particularly important to young companies that, while currently recording highly volatile returns, expect decreased volatility in the future. The lattice model allows for more precise assumptions, therefore allowing more precise estimates of option values. The lattice model uses data collected about employee exercise behavior and stock-price volatility to project an appropriate array of future exercise behaviors. This in turn allows more-accurate estimates of option values. In comparison to the Black-Scholes model, the lattice structure allows the incorporation of various early-exercise assumptions, once substantiated by an analysis of employee behavior patterns, which results in more-accurate, and often lower, option values and lower expenses.

The Logic of Lattice Models

Lattice-based option-pricing models, such as the binomial model, use estimates of expected stock-price movements over time. The expected magnitude and likelihood of stock-price movement is predicated upon the expected volatility of a security’s returns. Exhibit 1 illustrates a
simple two-year lattice model that depicts the expected price changes of the security, along with their probability of occurrence. Each node of the lattice reflects an expected year-end share price. These expectations are developed through analysis of the security's historical volatility and its likely future volatility.

Volatility
Volatility refers to the fluctuations in share returns over time. Volatility is measured by calculating the expected standard deviation of the returns of a security. The expected future volatility then determines expected share price movements over time. In turn, these potential share price movements are a major factor in estimating option value. Exhibit 1 illustrates how the estimated standard deviation results in share price movements over time.

The most common method of estimating future volatility is to use historical volatility as a proxy. There are no hard-and-fast guidelines on how far back one should calculate historical volatility. Future volatility can also be estimated by solving for the implied volatility of a company's traded options. Interpreting implied option volatility requires caution, as option volatility is impacted by the interaction of:
- The option's expected time to expiration,
- Whether the option is trading at-the-money, and
- General economic conditions.

Start-up companies generally have higher volatilities than do mature companies within an industry. Therefore, when calculating the volatility for a company that has been public for only a few years, its historical volatility may not be a good proxy for future volatility. When insufficient data exists to form estimates of a particular company's expected volatility, the FASB Exposure Draft Implementation Guide suggests using the volatility of comparable companies.

One advantage of a lattice model is that it can use different volatility estimates for different time periods. For example, options with a four-year expected life can have different expected volatilities over that period. The implied volatility of traded options with varying times to expiration can be used to estimate the various relevant implied volatilities. The Black-Scholes model requires the use of the same standard deviation over the entire expected life of an option, thereby reducing the flexibility of the model and the precision of the results.

Basic Example
Exhibit 2 illustrates an example with a 64.8% probability that the price of the security will increase 15% (from $30.00 to $34.50) and a 35.2% probability that the price will decline by 13% (from $30.00 to $26.09). The probabilities and percentage price increases are the same for each of the two years. For example, if the price does go up to $34.50 in year 1, there

Exhibit 2
Lattice with Option Values

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<th>Grant Date</th>
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The mechanics of calculating the option value at the time of grant begin by determining the option value at the expiration period and working backward to the date of the grant.

Exhibit 2 extends the analysis to illustrate how option values are determined. Assume that fully vested stock options have been granted with an exercise price of $30.00 and a term of two years. Therefore, the owner of the option can purchase shares of stock for $30.00 until the option expires in two years. If the share price increases in both years 1 and 2, the option holder will net $9.68 ($39.68 -- $30.00) upon exercise of the option. If the share price stays at $30.00 a share or falls to $22.58 at the end of year 2, the option holder will not exercise, as the share price does not exceed the exercise price.

If the share price has a value of $30.00 or less at the end of the two-year period, there is no gain for the holder, but there is also no loss. The option simply expires unexercised. At the time of the option grant, the option clearly has value. It is more likely that the stock will have a value greater than $30.00 at the end of two years, and the holder will not suffer any loss if it does not.

The mechanics of calculating the option value at the time of grant begin by determining the option value at the expiration period and working backward to the date of the grant. At the end of year 1, the share price will have either increased to $34.50 or fallen to $26.09. If the share price is $34.50 at the end of year 1, the option holder has an asset that will either rise $9.68 (share price of $39.68) or fall to $0 (share price of $30.00). The respective probability of these outcomes is 64.8% and 35.2%. Using a time value of money discount rate of 5%, the value of the option in year one will be $5.97, as calculated below:

\[
\frac{(64.8\% \times 9.68)}{1.05} + \frac{(35.2\% \times 0)}{1.05} = 5.97
\]

Continuing to work backward in time, the value of the option at the grant date is based upon the option values at the end of year 1. The calculation is the same as in the previous example, and yields an option value of $3.68, the present value...
value of $5.97 and $0 weighted by the probabilities of each outcome occurring:

\[(64.8\% \times 5.97) \div 1.05\] + \[(35.2\% \times 0) \div 1.05\] = $3.68

Thus, the option value is based upon the expected share price at each node on the lattice. If the historical volatility is higher, and the future volatility is projected to be higher, all else being equal, the option will have more value; the higher the probability of an increase in stock price, the higher the value of the option. There is no real risk of loss to the option holder, who will simply not exercise the option if the stock price declines. Therefore, as long as there is a positive probability that the price will rise above the exercise price, the option has value.

The analysis above illustrates the value of transferable options at the grant date. Employee stock options, however, are not transferable, and this affects their value.

The Transferability of Options
In the above example, if the share price has risen to $34.50, the option would be worth $5.97, factoring in the possibility of a rising price in year 2. But if the option cannot be sold, the option holder must choose between exercising the option at the end of year 1 and holding it until the end of year 2. If the holder chooses to exercise the option at the end of year 1, the proceeds would be only $4.50.

Because they cannot sell the option in the open market, many employees will exercise their options early to realize a gain rather than take the chance that the share price will fall. In other words, the option is worth only $5.97 at the end of year 1 if it can be sold. There is a positive probability that the stock will rise in year 2 and be worth $9.68, but it also might decline and become valueless. Employees may prefer to take a profit of $4.50 rather than risk losing all the potential value. The result of the potential early exercise is that the grant date value of the option falls from $3.68 to $2.78:

\[(64.8\% \times 4.50) \div 1.05\] + \[(35.2\% \times 0) \div 1.05\] = $2.78

The reduced option value is due to the increased likelihood of early exercise that nontransferability represents.

Early Exercise
Lattice-based structures allow expected employee exercise behavior to be modeled in order to develop more-accurate estimates of option values. Survey data indicate that many employees will exercise their options early to realize built-in gains when the underlying share price reaches 1.5 to 2.5 times the exercise price. (See Jennifer N. Carpenter, "The Exercise and Valuation of Executive Stock Options," Journal of Financial Economics, 1998.)

Exhibit 4 extends the example to a 10-period binomial with the expectation of early exercise when the underlying share price reaches 1.5 times the exercise price. As shown, when the price exceeds $45.00, employees will exercise their options, effectively stopping the binomial tree from expanding.

The blue cells indicate where early exercise takes place. The gray areas represent the portion of the binomial tree that

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is no longer relevant due to early exercise. This results in the option value falling from $12.34 to $8.83 as a result of early exercise. The loss in option value is due to the lost probability of further increases in share value. Data on the exercise history of employees can be incorporated into the exercise behavior of employee groups to generate a more-accurate and often lower option value than with the Black-Scholes option pricing model.

**Early Exercise with Vesting Requirements**

*Exhibit 4 adds an assumption of five-year cliff vesting.* Cliff vesting occurs when exercise of the option is not allowed until the end of a vesting period. This impacts the employee's ability to early exercise when the share-price-to-exercise-price ratio reaches 1.5. Vesting requirements extend the life of the option, increasing its value and resulting in added compensation expense. Exhibit 4 illustrates that the vesting requirements change the timing of employee early exercise. Even though the share price may reach $45.00 in year 3, early exercise cannot occur until vesting is allowed in year 5, as shown by the purple cells. The vesting requirement increases the expected time to exercise, thereby increasing the option value from $8.83 to $9.63.

One benefit of a lattice model is that it can analyze the trade-offs between the benefits of increased employee retention through longer vesting periods versus the added option expense of delaying early exercise. Other trade-offs, such as the reduced option cost of issuing options out-of-money, can be more fully understood using lattice structures.

**Early Exercise with Vesting Requirements and Forfeiture**

All companies experience option forfeiture due to layoffs or voluntary exit. Employees that exit prior to vesting do not impact the value of stock options, because the employee is not entitled to the option. Nonetheless, such exits impact the expected option expense through a reduction in the number of options expected to vest. Once the employee is vested, employee termination will lead to early exercise if the option is in-the-money.

The lattice tree shows how employee terminations impact the value of stock options. *Exhibit 5* shows the 10-year employee stock option with five-year cliff vesting and expected employee turnover of 3% annually for vested employees. Under the exercise rules in the previous example, the blue highlights show the cells where normal early exercise takes place. The cells in orange indicate additional early exercise nodes due to employee turnover exit. The value of the option decreases from $8.83 to $7.36 because of the additional early exercise.

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**Exhibit 5**

Early Exercise with Five-Year Cliff Vesting and Forfeiture

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