Local Bond Stress-Slip Models for Reinforcing Bars & Prestressing Strands in High-Performance Fiber Reinforced Cement Composites (HPFRCCs)

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Direct Tensile Test

Regular Concrete

FRC

HPFRCC

1 cm
Local Bond-Slip Test Setup and Loading Type

- **Monotonic Loading**
  - \( P \) (Load)
  - \( D \) (Displacement)

- **Unidirectional Cyclic**
  - \( P \) or \( D \)
  - Number of Cycle

- **Fully Reversed Cyclic**
  - \( P \) or \( D \)
  - Number of Cycle

- HPFRCC Prism
- Corner Plate
- Reinforcing Bar
Local Bond-Slip Test Setup and Loading Type

- Monotonic and Unidirectional Cyclic Loading
- Fully Reversed Cyclic Loading
Fibers

- Spectra (ultra high modulus polyethylene, UHMPE) Fiber
- Steel Hooked Fiber
- PVA 13 (polyvinyl alcohol) Fiber
- Twisted polygonal steel (Torex) Fiber
Typical Bond Stress-Slip Responses under Monotonic Loading (Reinforcing Bars)

Specimen with No. 25 Reinforcing Bar Matrix Compressive strength = 76 MPa

* With Spectra Fiber (2% Fiber Volume Fraction)

Average Bond Stress (MPa)

- HPFRCC*
- Spiral Reinforcement ($\rho_s = 2\%$)
- Plain Concrete

Slip (mm)
Fully Reversed Force-Controlled Cyclic Loading (Typical Results)

Plain Concrete

RC (2% Spiral)

HPFRCC (2% twisted steel fiber)
Reinforcing Bars - Performance Under Monotonic Loading

Typical Cracking Patterns

Regular Concrete

RC (2% Steel Spiral Reinforcement)

HPFRCC (2% Twisted Steel Fiber)
Typical Bond Stress-Slip Responses under Monotonic Loading (12.7 mm Seven-Wire Strand)

- Specimen with 12.7 mm dia. Strand
- Matrix Compressive strength = 76 MPa
- With Square Twisted Steel Fiber (1% Fiber Volume Fraction)
- HPFRCC*
- Spiral Reinforcement ($\rho_s = 2\%$)
- Plain Concrete

- Graph showing bond stress vs. slip for different materials.
Strands - Performance Under Reversed Cyclic Loading

- Plain Concrete: 9 full cycles
- RC (2% Spiral): 27 full cycles
- HPFRCC (2% Twisted Steel Fiber): 27 full cycles
Typical Bond Stress-Slip Models for Reinforcing Bars embedded in FRCC:

- **Ascending branch**:
  \[ \tau(s) = \tau_{\text{max}} \left( \frac{s}{s_{\text{max}}} \right)^{0.5} \]

- **Descending branch**:
  \[ \tau(s) = \tau_f \left( \frac{s}{s_f} \right) \left( \frac{s_{\text{max}} - \tau_f}{s_{\text{max}} - s_f} \right) \]

\[ \tau_{\text{max}} = \left( 1.18 + 3.73 \frac{c}{d_b} + 22.38 \frac{d_b}{L_e} \right) \left( \sigma_r \right)^{0.8} \text{ (MPa)} \]

- \( \tau_f \approx 0.45 \tau_{\text{max}} \)
- \( s_{\text{max}} = 1.3 \text{ mm} \)
- \( S_f = 10.2 \text{ mm} \)

Note: \( L_e \) = embedment length; \( c \) = concrete cover; \( d_b \) = bar diameter; \( \sigma_r \) = flexural strength of SIFCON matrix

Reinforcing Bars in SIFCON (Hamza, 1992)
Typical Bond Stress-Slip Models (continued):

Reinforcing Bars in SFRC (Harajli, 2002)

Ascending branch: $\tau(s) = \tau_{\text{max}} \left(\frac{s}{s_1}\right)^{0.3}$ if $0 \leq s \leq s_1$

$\tau_{\text{max}}$ (MPa) = $2.57\sqrt{f_c'}$ (MPa)

$\tau_{\text{splitting}} = c_f \left(0.75\sqrt{f_c'} \left(c / d_b\right)^{2/3}\right) \leq \tau_{\text{max}}$

where $V_fL / d_f \leq 0.25 \Rightarrow c_f = 1.0$

$V_fL / d_f > 0.25 \Rightarrow c_f = 1 + 0.34\sqrt{V_fL / d_f} - 0.25$

$\tau_{ps} = \left[0.33 + 0.37 \left(c / d_b\right) \left(V_fL / d_f\right)\right]\sqrt{f_c'} \leq \tau_{\text{splitting}}$

$\tau_f = 0.35\tau_{\text{max}}$

$S_{\text{splitting}} = S_1 e^{1.8\left(\frac{\tau_{\text{splitting}}}{\tau_{\text{max}}} - 1\right)}$

$S_1 = 1.5 \text{ mm}$  $S_2 = 3.5 \text{ mm}$  $S_3 = 10 \text{ mm}$
Local Bond Stress-Slip Modeling based on Tensile Stress-Strain Responses of (Proposed)

Direct Tensile Test

Minimum information on tensile strain-hardening stress-strain response of FRCCs needed for design and modeling (Naaman and Reinhardt, 2006)
Separation-Type Failure Mode

Half-Specimen

Bridging Stress

\[ \sigma_{pc} \]

Crack surface

\[ \sigma_{cc} \]

Reinforcing Bar

\[ f_r \]

\[ d_b \]

\[ L \]

Strut

Pull End

\[ F \]

\[ F_t \]

\[ 50.0^\circ \]
Proposed Local Bond Stress-Slip Model for Reinforcing Bar embedded in HPFRCC

### Ascending branch
\[ \tau(s) = \tau_{\text{max}} \cdot \left( \frac{s}{s_{\text{max}}} \right) \]

### 0.2\% \leq \varepsilon_{pc} \leq 0.6\%

\[ \tau_{\text{max}} = 0.5 \cdot \left( \frac{c}{\eta \cdot d_b} \right) \cdot \left( \sigma_{cc} + \sigma_{pc} \right) - \frac{\sigma_{cc} \cdot \sigma_{pc}}{E_c \cdot \varepsilon_{pc}} \quad \text{(Mpa)} \]

\[ \tau_f = 0.15 \tau_{\text{max}} \quad \text{(Mpa)} \]

### Diagrams
- **Bond Stress vs. Slip**
  - **\( \tau_{\text{max}} \)**
  - **\( \tau_f \)**
  - **\( S_{\text{max}} \)**
  - **\( S_f \)**

- **Tensile Stress vs. Strain**
  - **\( \sigma_{cc} \)**
  - **\( \sigma_{pc} \)**
  - **\( E_c \)**
  - **\( \varepsilon_{cc} \)**
  - **\( \varepsilon_{pc} \)**
Interface-Crushing-Type Failure Mode

\[ \sigma_c \]

Clamping stress

Crack surfaces
Proposed Local Bond Stress-Slip Model for Reinforcing Bar embedded in HPFRCC

\[ \tau_{\text{max}} = 40 \cdot \left( \frac{\sigma_{pc} \cdot (f'\_c)^{1/4}}{\eta \cdot d_b} \right) \text{(Mpa)} \]

\[ \tau_f = 0.3 \tau_{\text{max}} \text{(Mpa)} \]

\[ S_{\text{max}} = \frac{215 \cdot \tau_{\text{max}}}{d_b \cdot f'\_c} \text{(mm)} \]

\[ S_f = 12.7 \text{ mm} \]
Proposed Local Bond Stress-Slip Model for Seven-Wire Strand embedded in HPFRCC

Ascending branch: \( \tau(s) = \frac{\tau_{\text{max}} \cdot S}{s_{\text{max}}} \)

- \( \varepsilon_{pc} \geq 0.6\% \) (in order to maintain the tensile capacity after cracking occurs)

- \( \tau_{\text{max}} = 0.275 \cdot \left[ \frac{\sigma_{cc} \cdot f_c'}{d_b} \right] \) (MPa)
- \( s_{\text{max}} = \tau_{\text{max}} / 4 \)
- \( s_f = 20 \text{ mm} \)
Local bond stress-slip models are proposed for reinforcing bars and prestressing strands embedded in HPFRCC. The proposed models were derived based on tensile stress-strain characteristics of HPFRCC (composite elastic modulus, first percolation cracking stress and its corresponding strain, and peak post-cracking stress and its corresponding strain), where a tensile strain-hardening response occurs up to large strains.

Further verification of the proposed models by using other types (such as beam-type) of specimens is suggested.
A STATISTICAL THEORY OF STRENGTH
FOR FIBER REINFORCED CONCRETE

by

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Publications:

(with S. P. Shah) "Tensile Tests of Ferro-Cement," Journal of the American Concrete Institute, September 1971.

