PERFORMANCE-BASED PLASTIC DESIGN (PBPD) METHOD FOR EARTHQUAKE-RESISTANT STRUCTURES: AN OVERVIEW

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SUMMARY

This paper presents a brief overview of performance-based plastic design method as applied to the seismic design of building structures. The method uses pre-selected target drift and yield mechanisms as key performance criteria. The design base shear for a selected hazard level is calculated by equating the work needed to push the structure monotonically up to the target drift to that required by an equivalent single degree of freedom to achieve the same state. Plastic design is performed to detail the frame members and connections in order to achieve the targeted yield mechanism and behaviour. The method has been successfully applied to a variety of common steel framing systems and, more recently, to Reinforced Concrete (RC) moment frames. Results of extensive inelastic static and dynamic analyses showed that the frames developed desired strong column-sway mechanisms, and the storey drifts and ductility demands were well within the target values, thus meeting the desired performance objectives. The examples of 20-storey steel and RC moment frames, as presented in the paper, showed that the method is especially advantageous for tall frames, where cumbersome and lengthy iterative design work in current practice can be completely eliminated, while leading to excellent performance as targeted. The basic work-energy equation can also be used for seismic evaluation of existing structures. The results, as presented in this paper, showed excellent agreement with those obtained from more elaborate inelastic time-history analyses. Copyright © 2009 John Wiley & Sons, Ltd.

1. INTRODUCTION

It is well known that structures designed by current codes undergo large inelastic deformations during major earthquakes. However, current seismic design approach is generally based on elastic analysis and accounts for inelastic behaviour in a somewhat indirect manner. In the current US seismic design practice, it is common to obtain design base shear from code-specified spectral acceleration, assuming the structures to behave elastically, and reducing it by force reduction factor, \( R \), depending upon available ductility of the structural system. The design forces are adjusted for the importance of specific structures by using an occupancy importance factor, \( I \). After selecting the member sizes for required strengths, as obtained from elastic analysis, the calculated drift at design forces is multiplied by a deflection amplification factor, \( C_d \), and kept within specified limits (generally in the order of 2%). Appropriate detailing provisions are followed in order to meet the expected ductility demands. When struck by severe ground motions, however, the structures designed by such procedures have been found to undergo inelastic deformations in a somewhat ‘uncontrolled’ manner. The inelastic activity, which may include severe yielding and buckling of structural members and connections, can be

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unevenly and widely distributed in the structure. This may result in a rather undesirable and unpredictable response, sometimes total collapse, or difficult and costly repair work at best.

While the above design practice has served the profession rather well in the past, societal demands are pushing the practice to achieving higher levels of performance, safety and economy, including life-cycle costs. For the practice to move in that direction, design factors, such as determination of appropriate design lateral forces and member strength hierarchy, selection of desirable yield mechanism, structure strength and drift, etc., for specified hazard levels should become part of the design process right from the start.

Practical design methods are needed to achieve the above-mentioned goals. One such complete design methodology, which accounts for inelastic structural behaviour directly, has been developed by the senior author and his associates at the University of Michigan (Lee et al., 1999, 2007; Lee and Goel, 2001; Dasgupta et al., 2004; Chao and Goel, 2006a, 2006b, 2008; Chao et al., 2007; Goel and Chao, 2008). The design concept uses pre-selected target drift and yield mechanisms as key performance objectives. Results of extensive inelastic static and dynamic analyses have proven the validity of the method. The method has been successfully applied to steel Moment Frame (MF), buckling restrained braced frame (BRBF), Eccentrically Braced Frame (EBF), Special Truss Moment Frame (STMF), and concentric braced frames. In all cases, the frames developed the desired strong column–weak beam yield mechanisms as intended, and the storey drifts/ductility demands were well within the selected design values, thus meeting the selected performance objectives. Comparisons of responses with corresponding baseline frames designed by current practice have consistently shown superiority of the proposed methodology in terms of achieving the desired behaviour.

Development of the performance-based plastic design (PBPD) method for RC building structures is currently in progress. An overview of the method and examples of a 20-storey steel moment frame and a 20-storey RC moment frame are briefly presented in the paper.

2. BACKGROUND

Use of energy equation in a simple form along with limit state design was first proposed by Housner (1956). Housner used the difference between the input energy and the elastic strain energy to obtain the plastic energy to be ‘absorbed’ by the structure as a means to design the intended yielding members. In the example of a steel-braced tower for water tank, the tension-only bracing rods were designed to have adequate plastic energy capacity up to the ultimate strain. In a follow-up paper, Housner (1960) extended the concept to determine the design lateral force to prevent collapse due to overturning at extreme drift. Examples included a simple cantilever column supporting a heavy mass, such as a water tower, and a multi-storey building structure. Due to simplicity of the approach and limited available knowledge at that time, a number of assumptions were made.

Housner (1960) observed that during a strong ground shaking, structures may fail in one of several ways:

One possibility is that the vibrations will cause approximately equal plastic straining in alternate directions and that this will continue until the material breaks because of a fatigue failure. Another possibility is that all of the plastic straining will take place in one direction until the column collapses because of excessive plastic drift. These two possibilities are extreme cases, and the probability of their occurrence is small. The most probable failure is collapse due to greater or lesser amount of energy having been absorbed in plastic straining in the opposite direction. In this case collapse occurs when some fraction of the total energy pE is just equal to the energy required to produce collapse by plastic drift in one direction. In what follows, the factor p will be taken equal to unity as a matter of convenience, . . . .
The energy concept used in the development of the PBPD method is very similar to the basic approach used by Housner about 50 years ago. In the PBPD method, the relationship between the amount of work needed to push the structure monotonically up to the design target drift and the elastic input energy is derived on a rational basis by using inelastic response spectra for elastic-plastic single degree of freedom (EP-SDOF) systems. The method is extended to multi-storey structures through equivalent modal SDOF oscillators.

3. PBPD METHOD

As mentioned earlier, the PBPD method uses pre-selected target drift and yield mechanisms as key performance objectives. These two design parameters are directly related to the degree and distribution of structural damage, respectively. The design base shear for a specified hazard, which is generally given as design spectrum in the codes, is calculated by equating the work needed to push the structure monotonically up to the target drift to the energy required by an equivalent EP-SDOF to achieve the same state (Figure 1). Also, a new distribution of lateral design forces is used that is based on relative distribution of maximum storey shears consistent with inelastic dynamic response results (Chao et al., 2007). Plastic design is then performed to detail the frame members and connections in order to achieve the intended yield mechanism and behaviour. Thus, determination of design base shear, lateral force distribution and plastic design are three main components of the PBPD method, which will be discussed in the following sections.

It should be noted that in this design approach, the designer selects the target drifts consistent with acceptable ductility and damage, and a yield mechanism for desirable response and ease of post-earthquake damage inspection and reparability. The design lateral forces are determined for the given seismic hazard (design spectrum) and selected target drift. Thus, there is no need for factors, such as $R$, $I$, $C_d$, etc., as required in the current design codes and over which plenty of debates already exist.

![Figure 1. PBPD concept](image-url)
Those factors are known to be based on a number of considerations including engineering judgement.

It is important to note that in the PBPD method, control of drift and yielding is built into the design process from the very start, eliminating or minimizing the need for lengthy iterations to arrive at the final design. Other advantages include the fact that innovative structural schemes can be developed by selecting suitable yielding members and/or devices and placing them at strategic locations, while the designated non-yielding members can be detailed for no or minimum ductility capacity. All of these would translate into enhanced performance, safety and economy in life-cycle costs.

3.1 Design base shear

Determination of the design base shear for a given hazard level is a key element in the PBPD method. It is calculated by equating the work needed to push the structure monotonically up to the target drift to that required by an equivalent EP-SDOF system to achieve the same state. Assuming an idealized EP force-deformation behaviour of the system, the work-energy equation can be written as:

\[
(E_e + E_p) = \gamma \left( \frac{1}{2} MS^2 \right) = \frac{1}{2} \gamma M \left( \frac{T}{2\pi} S_a g \right)^2
\]

where \(E_e\) and \(E_p\) are, respectively, the elastic and plastic components of the energy (work) needed to push the structure up to the target drift; \(S\) is the design pseudo-spectral velocity; \(S_a\) is the pseudo-spectral acceleration; \(T\) is the natural period; and \(M\) is the total mass of the system. The energy modification factor, \(\gamma\), depends on the structural ductility factor (\(\mu_s\)) and the ductility reduction factor (\(R\mu\)), and can be obtained by the following relationship:

\[
\gamma = \frac{2\mu_s - 1}{R\mu^2}
\]

Because of their simplicity, spectra proposed by Newmark and Hall (1982), as shown in Figure 2, were used to relate the ductility reduction factor, \(R\mu\), and the structural ductility factor, \(\mu_s\), for EP-SDOF. Plots of energy modification factor, \(\gamma\), as obtained from Equation (2), are shown in Figure 3 (Lee and Goel, 2001). It should be mentioned that for this purpose, any inelastic spectra for EP-SDOF systems can be used as preferred, such as that by Miranda and Bertero (1994).
The work-energy equation can be rewritten in the following form:

\[
\frac{1}{2} \left( \frac{W}{g} \right) \times \left( \frac{T}{2\pi} \times \frac{V_y}{W} \right)^2 + V_y \left( \sum_{i=1}^{N} \lambda_i h_i \right) \theta_p = \frac{1}{2} \gamma \left( \frac{W}{g} \right) \times \left( \frac{T}{2\pi} S_a g \right)^2
\]

or

\[
\left( \frac{V_y}{W} \right)^2 + \frac{V_y}{W} \left( h^* \times \frac{\theta_p 8\pi^2}{T^2 g} \right) \theta_p - \gamma S_a^2 = 0
\]

The admissible solution of Equation (4) gives the required design base shear coefficient \( V_y/W \):

\[
\frac{V_y}{W} = \frac{-\alpha + \sqrt{\alpha^2 + 4\gamma S_a^2}}{2}
\]

where \( \alpha \) is a dimensionless parameter given by

\[
\alpha = \left( h^* \times \frac{\theta_p 8\pi^2}{T^2 g} \right)
\]

The term \( \theta_p \) represents the plastic component of the target drift ratio, and \( h^* = \sum_{i=1}^{N} (\lambda_i h_i) \).

As mentioned earlier, Equation (5) for \( V_y \) was derived by assuming ideal EP force-deformation behaviour and ‘full’ hysteretic loops for the system. That is characteristic of a number of ductile steel framing systems, such as MF, EBF, STMF and BRBF. For systems that do not possess such hysteretic property, such as steel-braced frames with buckling type braces or RC frames, some modification is warranted. Two approaches have been tried that show good promise.

(1) The energy capacity term, represented by the left-hand side of Equation (1), can be modified by a factor \( \eta \) to account for the reduced area of typical hysteretic loops as a fraction of the corre-
sponding ‘full’ loops (Figure 4). Thus, Equations (1) and (5) can be modified as Equations (7) and (8), respectively:

\[ \eta(E_s + E_p) = \gamma \left( \frac{1}{2} M S_e^2 \right) = \frac{1}{2} \gamma M \times \left( \frac{T}{2\pi} S_{eg} \right)^2 \]  

\[ \frac{V_t}{W} = -\alpha + \frac{\alpha^2 + 4(\gamma/\eta) S_u^2}{2} \]  

(2) The second approach is based on consideration of the effect of degrading hysteretic behaviour on peak (target) displacement. Investigators have studied the effect of degrading hysteretic behaviour of SDOF systems on resulting peak displacements. The results show that the peak displacements are larger than those of systems with non-degrading hysteretic behaviour in the short period range, but are about equal for longer periods. Approximate expressions have been proposed for modification factors to account for this effect, e.g. factor \( C_2 \) in FEMA (2006) (Figure 5). Thus, the target design drift for a given structural system with degrading hysteretic behaviour can be divided by the \( C_2 \) factor, which would give design target drift for an equivalent non-degrading
system. The design base shear can then be calculated by using this modified target drift and Equation (5).

3.2 Lateral force distribution

Traditionally, vertical distribution of design lateral forces in modern building codes has been based on elastic fundamental mode response of Multi-Degree-of-Freedom (MDOF) systems, resulting in the following basic expression for force at level \( i \):

\[
F_i^* = \left( \frac{w_i h_i}{\sum_{j=1}^{n} w_j h_j} \right) V
\]

where the terms \( w \) and \( h \) represent seismic weight and height above the base, respectively, and \( V \) represents the total base shear. Summation of lateral forces above the level \( i \) will represent the shear force, \( V_i^* \), at that level. Thus,

\[
V_i^* = \left( \frac{\sum_{j=1}^{n} w_j h_j}{\sum_{j=1}^{n} w_j h_j} \right) V
\]

Modifications have been made in the above basic equations to account for effects, such as contribution of higher modes. Most recently, the International Code Council (ICC) (2006) prescribes the following equation for design lateral force at level \( i \):

\[
F_i^* = \left( \frac{w_i h_i^k}{\sum_{j=1}^{n} w_j h_j^k} \right) V
\]

where the exponent \( k \) is a function the fundamental period. However, results of inelastic dynamic response studies carried out on a number of structural systems showed that the distributions of maximum storey shears along the height were quite different from those given by the codes (Chao and Goel, 2007). Figure 6 shows a sample of such results, where the storey shear at level \( i \) is plotted as ratio of that at the top \( n^{th} \) level. In order to bring the design storey shear distribution closer to the inelastic response results, an exponent term was used, as shown in the following equation:

\[
V_i = \left( \frac{\sum_{j=1}^{n} w_j h_j^{0.75T^{-0.2}}}{{\sum_{j=1}^{n} w_j h_j}} \right)^{0.75T^{-0.2}} V_y
\]

The constants in the exponent term in Equation (12) were derived by empirical fit with inelastic response results of a variety of common structural systems (Chao and Goel, 2007). \( V_y \) represents the
design base shear as determined from Equation (5) or Equation (8) for use in the PBPD method. For \( i = n \), Equation (12) gives the value of shear, \( V_n \), or lateral force, \( F_n \), at the top level:

\[
V_n = F_n = \left( \frac{w_i h_n}{\sum_{j=1}^{n} w_j h_j} \right)^{0.75 T^{-0.02}} V_y
\]  

(13)

Combining Equations (12) and (13), the ratio \( V_i / V_n \), termed as shear distribution factor, \( \beta \), can be expressed as:

\[
\frac{V_i}{V_n} = \beta_i = \left( \frac{\sum_{j=1}^{n} w_j h_j}{w_i h_n} \right)^{0.75 T^{-0.02}} V_y
\]  

(14)

Then, the lateral force at level \( i \), \( F_i \), can be obtained as

\[
F_i = (\beta_i - \beta_{i+1}) V_n
\]  

(15)

or

\[
F_i = (\beta_i - \beta_{i+1}) \times \left( \frac{w_i h_n}{\sum_{j=1}^{n} w_j h_j} \right)^{0.75 T^{-0.02}} V_y
\]  

(16)
4. DESIGN EXAMPLE 1: 20-STOREY STRUCTURAL ENGINEERS ASSOCIATION OF CALIFORNIA (SEAOC), THE APPLIED TECHNOLOGY COUNCIL (ATC), AND CONSORTIUM OF UNIVERSITIES FOR RESEARCH IN EARTHQUAKE ENGINEERING (CUREE) STEEL MOMENT FRAME

Moment frames are very common for steel as well as RC building structures. An example of a tall steel moment frame is presented in this section. This 20-storey frame was the subject of an extensive analytical study as part of the SAC steel research programme (Gupta and Krawinkler, 1999). The frame was redesigned by the PBPD method as outlined in the preceding sections. The resulting design and its responses under static pushover and dynamic time-history analyses due to selected set of ground motions are compared with those of the original SAC frame.

It should be mentioned that in current practice, the design of moment frames is generally governed by drift limits. That makes the design process highly iterative, especially for tall frames. After the initial member sizes are selected on the basis of required strength, extensive revisions are needed in order to satisfy the prescribed drift limits and other requirements, such as strong column–weak beam proportions. This is carried out in a purely trial-and-error manner, without any guidance provided to the designer. In contrast, the PBPD method is a direct design method where drift and yield mechanisms, e.g. strong column–weak beam condition, are built in the design process from the very start. Thus, iterative work is reduced to a very minimum effort, if not totally eliminated. This advantage becomes particularly important for tall frames where the iterative process in current practice can be very cumbersome and lengthy, requiring checking by extensive inelastic pushover and/or time-history analyses.

The framing plan of the structure is shown in Figure 7. Since the original SAC frame was designed according to the Uniform Building Code (UBC) (1994), same loading and other design parameters were used for the redesigning of the frame by PBPD method. The storey heights are 18 ft for the first storey and 13 ft for all others. The seismic floor weights are shown in Table 1.

4.1 Design base shear and lateral force distribution

The target yield mechanism is similar to the one shown in Figure 8 with beam plastic hinges assumed at 6 in. from the column face. The shear distribution factor and design lateral force distribution are
calculated from Equations (14) and (16). According to the UBC (1994), the elastic design spectral acceleration, \( S_a = Z I C \), where \( Z \) is the seismic zone factor, \( I \) is the occupancy importance factor and \( C \) is the seismic coefficient, is to be calculated as follows:

\[
C = 0.075 R_u \leq 1.25 S/T^{2/3} \leq 2.75
\]  

(17)

With \( S = 1.2 \) for S2 soil type, \( Z = 0.4 \), \( I = 1.0 \) and estimated \( T = 2.299 \) s, the value of \( S_a \) turned out to be equal to 0.36.

The design base shear was determined for two level performance criteria: (1) A 2% maximum storey drift ratio \( (\theta_u) \) for ground motion hazard with a 10% probability of exceedance in 50 years (10/50 or 2/50 maximum considered earthquake (MCE)); (2) A 3% maximum storey drift ratio \( (\theta_u) \) for 2/50 or MCE hazard. A yield drift ratio \( (\theta_y) \) of 1.0% was used, which is typical for steel moment frames. The calculated values of significant design parameters are listed in Table 2. It can be seen that the design base shear for the first hazard level (2/3 MCE) governs.

4.2 Design of beams (designated yielding members)

When using the target yield mechanism for moment frames, as shown in Figure 8, beams become the primary designated yielding members (DYM). The required beam moment capacity at each level
can be determined by plastic design approach (external work equals internal work) and referring to Figure 8:

\[
\sum_{i=1}^{n} F_i h_i \theta_i = 2M_{pb} \theta + \sum_{i=1}^{n} 2(\beta_i M_{pb}) \gamma_i
\]  

(18)

where \( \theta \) represents a small kinematic rotation angle of the yield mechanism, and \( M_{pb} \) and \( \beta M_{pb} \) are the required moment strengths at the top floor level and level \( i \), respectively. The rotation term \( \gamma = (L/L_i) \) (Figure 8). It is noted that the external work carried out by uniformly distributed gravity loading is zero due to antisymmetrical deformed shape of the beams.

Previous studies have shown that it is desirable to have the distribution of structural strength along the building height follow the distribution of design storey shears, i.e. \( M_{ph} = \beta M_{pb} \). This helps to distribute the yielding more evenly along the height, thereby preventing yielding from concentrating at a few levels. Thus, the number of unknown terms in Equation (18) is reduced to two, i.e. \( M_{pb} \) and \( M_{pc} \). By assuming a suitable value of \( M_{pc} \), the required value of \( M_{pb} \) (and thus \( M_{ph} \)) can be obtained by solving Equation (18). The selection of appropriate beam sections can then be performed by using applicable specification.

Alternatively, the value of \( M_{pc} \) can be determined by using the condition that no soft storey mechanism would occur in the first storey when a factor \( \Psi \) (in the order of 1·1 to 1·5) times the design lateral forces is applied on the frame, as shown in Figure 9. Assuming that plastic hinges form at the base and top of the first storey columns, the corresponding work equation for a small mechanism deformation, \( \theta \), gives

\[
M_{pc} = \frac{\Psi V' h_1}{4}
\]  

(19)

where \( V' \) is the base shear (for an equivalent one bay model), which may be taken as \( V \) divided by the number of bays; \( h_1 \) is the height of the first storey; and the factor \( \Psi \) accounts for overstrength above the design force. This method was used in the design example with a value of 1·5 for the overstrength factor \( \Psi \).

### Table 2. Design parameters for a 20-storey steel PBPD frame

<table>
<thead>
<tr>
<th>Design parameters</th>
<th>10% in 50 years hazard (2/3 MCE)</th>
<th>2% in 50 years hazard (MCE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_a )</td>
<td>0·36 g</td>
<td>0·54 g</td>
</tr>
<tr>
<td>( T )</td>
<td>2·299 s</td>
<td>2·299 s</td>
</tr>
<tr>
<td>Yield drift ratio ( \theta_y )</td>
<td>1·0%</td>
<td>1·0%</td>
</tr>
<tr>
<td>Target drift ratio ( \theta_u )</td>
<td>2%</td>
<td>3%</td>
</tr>
<tr>
<td>Inelastic drift ratio ( \theta_p = \theta_u - \theta_y )</td>
<td>1%</td>
<td>2%</td>
</tr>
<tr>
<td>( \mu = \theta_p / \theta_y )</td>
<td>2·0</td>
<td>3·0</td>
</tr>
<tr>
<td>( R_\mu )</td>
<td>2·0</td>
<td>3·0</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>0·75</td>
<td>0·438</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0·942</td>
<td>1·884</td>
</tr>
<tr>
<td>( V/W )</td>
<td>0·094</td>
<td>0·037</td>
</tr>
<tr>
<td>Design base shear ( V' )</td>
<td>1146 kips (governs)</td>
<td>457 kips</td>
</tr>
</tbody>
</table>

*For one moment frame in N–S direction.*
4.3 Design of columns (non-DYM)

Members that are not designated to yield (non-DYM), such as columns in an MF, must have design strength to resist the combination of factored gravity loads and maximum expected strength of the DYM by accounting for reasonable strain-hardening and material overstrength. For this purpose, one approach is to consider the equilibrium of portions of the design yield mechanism in the extreme limit state. In building frames, these portions include columns that can be modelled as ‘column trees’. For example, Figure 10 shows the free-body diagram of an exterior ‘column tree’ of the frame at the target drift. In order to ensure the formation of an intended strong column–weak beam mechanism, the columns must be designed for maximum expected forces by including gravity loads on beams and columns, and by considering a reasonable extent of strain-hardening and material overstrength in the beam plastic hinges. The columns at the base are also assumed to have reached their maximum capac-
ity, $M_{pc}$. The moment at a strain-hardened beam plastic hinge can be obtained by multiplying its nominal plastic moment ($M_{pb}$) by an appropriate overstrength factor ($\xi$), which accounts for the effect of strain-hardening and material overstrength. At this stage, the required lateral forces ($F_{iu}$) acting on this free body may be assumed to maintain the distribution as given by Equation (16), and their magnitude can be easily obtained by using equilibrium of the entire free body. Then the column end moments and shear force in each storey are calculated by applying the expected beam end moments and lateral forces ($F_{iu}$) applied at each level.

An important factor that amplifies the moments in the design of columns, as non-DYM, is P-Delta effect. This effect needs to be accounted for in the design process. One of the following methods can be used for this purpose:

4.3.1 Use of moment amplification factors
Column moments calculated from first-order analysis of the ‘column trees’ are amplified by using factors, such as B1 and B2 as given in the AISC (2005).

4.3.2 Direct P-Delta consideration in column tree analysis
In this method, which can be considered a more direct way of considering P-Delta effect, the ‘column tree’ is considered in an assumed deflected shape at target drift. A linear deflected shape is assumed herein. The gravity loads can be applied directly on the ‘column tree’ or on a ‘P-Delta column’, which is modelled for this purpose as shown in Figure 10. Equilibrium equation of the assembly is formulated to calculate the total required lateral force, $F_L$, and the resulting column moments and shears are obtained.

4.3.3 Pushover analysis of the frame
In this approach, a nonlinear static pushover analysis of the entire frame is carried out up to the target drift by using design lateral force distribution. The DYM are modelled to behave inelastically, while the non-DYM are modelled (or ‘forced’) to behave elastically. P-Delta effect is captured by applying the floor gravity loads on ‘gravity columns’ (columns not part of the lateral force resisting frame), which can be lumped into one.

Column moments for the example 20-storey steel PBPD frame were calculated by using the above three methods, and the results are shown in Figure 11. Also plotted in the figure are maximum column moments obtained from inelastic dynamic analyses by using SAC ground motion records representing 10/50 and 2/50 hazard levels. It can be seen that the column tree analysis results, amplified by B2-factor, give the most appropriate design moments for columns in all storeys, while also being on the safe side. Therefore, this method was used to design the columns of the example frames in this study.

4.4 Redesign of 20-storey SAC frame by PBPD method
The PBPD procedure, as described in the preceding sections, was used to redesign the 20-storey SAC frame. The target yield mechanism is similar to the one shown in Figure 8 with the beam plastic hinges assumed at 6 in. from the column face, assuming ductile unreinforced flange connections with the columns. Grade 50 steel ($F_y = 50$ ksi) was used. Table 3 shows comparison of material weights of the SAC and PBPD frames. More details can be found elsewhere (Goel and Chao, 2008).
4.5 *Inelastic response analysis results of the PBPD and SAC frames*

Nonlinear static (pushover) and dynamic (time-history) analyses were carried out for the SAC and PBPD frames by using PERFORM-3D programme (CSI, 2007). A lumped ‘gravity column’ was added that enables the model to capture the P-Delta effect as well as the contribution of all gravity columns in the response of the structure. Strength degradation of moment-rotation behaviour of plastic hinges and panel zone deformations were also modelled.

Figure 12 shows base shear versus roof drift plot of the two frames from the pushover analysis. It can be noticed that the PBPD frame has a somewhat greater ultimate strength than the SAC frame. The design base shears for the two frames are also shown in Figure 12. It is worth noting that the design base shear for the SAC frame is significantly smaller than its ultimate strength. That is because for moment frames, the code-specified design base shear is generally not adequate for drift control. For both frames, sharp decrease in strength after reaching the ultimate strength is especially notable.

Table 3. Material weights for one steel moment frame (five bays)

<table>
<thead>
<tr>
<th>Weight calculation</th>
<th>PBPD frame (kips)</th>
<th>SAC frame (kips)</th>
<th>PBPD/SAC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beams</td>
<td>178.4</td>
<td>191.8</td>
<td>0.93</td>
</tr>
<tr>
<td>Columns</td>
<td>398.0</td>
<td>318.9</td>
<td>1.25</td>
</tr>
<tr>
<td>Total</td>
<td>576.4</td>
<td>510.7</td>
<td>1.13</td>
</tr>
</tbody>
</table>

Figure 11. Interior column moments from inelastic dynamic analysis compared with different design methods
This is clearly because of the more prominent role of P-Delta effect in tall frames as compared with shorter frames. Figure 13 shows plots of deformed shape and plastic hinge formation in the two frames. As targeted in the design, absence of plastic hinges in columns of the PBPD frame (except at the bases) can be clearly noticed. In contrast, significant plastic hinging in the columns of the SAC frame in lower storeys is observed, which is also reflected in the differences in deformed shape of the two frames, i.e. concentration of drift in the lower storeys of the SAC frame. Also, plastic rotations in the beams and column bases at 3-5% roof drift are significantly smaller in the PBPD frame as compared with those in the SAC frame.

Inelastic dynamic analyses of both frames were carried out by using 10/50 and 2/50 SAC ground motions for Los Angeles site (Somerville et al., 1997). The results showed very good behaviour of the PBPD frame under 10/50 as well as 2/50 hazard levels. No unexpected plastic hinging was observed in the columns of the PBPD frame, whereas significant plastic hinging was observed in the SAC frame columns under both sets of ground motions (Goel and Chao, 2008). Moreover, collapse of the SAC frame was seen for three ground motions, namely LA30, LA35 and LA36. Even though those ground motions caused large storey drifts in the PBPD frame, the structure did not lose stability (Figure 14).

5. EVALUATION

The energy-based PBPD method, thus far, has been presented and discussed in the context of design of new structures for a target maximum drift. Therefore, with other terms being known, the design base shear is determined by solving the work-energy Equation (1). It turns out that the same energy equation can also be used for evaluation purposes, where the structure is defined, including
its force-displacement characteristics, and the goal is to ‘predict’ the expected maximum displacements for a given seismic hazard (Leelataviwat et al., 2007). Other response quantities, such as component forces and deformation demands, can then be easily calculated from the maximum reference displacement.
In order to use the energy concept for evaluation purposes, the right-hand side of Equation (1) can be viewed as energy demand for the given hazard, $E_d$, and the left-hand side can be viewed as energy capacity of the given structure, $E_c$. Both these quantities vary with displacement. The value of the desired maximum reference displacement can be obtained by either solving the work-energy equation analytically, or graphically by constructing the two energy curves as a function of the reference displacement and determining their point of intersection.

Figure 15 presents a graphical illustration of the evaluation process. Lateral force-displacement plot for the given structure is shown in Figure 15(a), where $V$ represents the total force (base shear) and $u_r$ represents the roof displacement, used as reference displacement. This plot can be obtained by a static pushover analysis by applying either an appropriately selected force or displacement pattern. It is common to plot total force versus roof displacement, but it can be carried out for any other floor or storey level from which the force or displacement at other levels can be determined. The energy capacity curve, $E_c - u_r$, can be generated as a function of $u_r$ by calculating the work carried out by lateral forces up to the displacement at each level corresponding to $u_r$ (Figure 15(b)). Next, the energy demand, $E_d$, can be calculated for varying values of $u_r$, and plotted as shown in Figure 15(c). The point of intersection of the two curves, where the energy demand and capacity become equal, gives the desired maximum roof displacement, as shown in Figure 15(d).

6. DESIGN EXAMPLE 2: 20-STOREY RC MOMENT FRAME

RC structures present special challenge due to their complex and degrading (‘pinched’) hysteretic behaviour. While development of the PBPD method for RC structures is currently in progress, results from the study so far have been most promising. An example of a 20-storey RC moment frame structure is briefly presented in this section. This baseline space frame was designed to comply with

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Figure 15. Proposed energy-based evaluation method for MDOF systems: (a) pushover curve; (b) energy-displacement capacity diagram; (c) energy demand diagram; and (d) determination of displacement demand
the requirements of ASCE 7-05 and ACI 318R-05 (Haselton and Deierlein, 2007; FEMA, 2009a) (ASCE, 2005; ACI, 2005). It was redesigned by the modified PBPD method by using the FEMA (2006) C2 factor approach as discussed in Section 3.1. Typical floor plan is shown in Figure 16, and important design parameters are given in Table 4. For response evaluation purposes, the baseline code compliant frame and the PBPD frame were subjected to inelastic pushover and time-history analyses. PERFORM-3D computer programme was used.

Table 4. Design parameters for PBPD 20-storey RC moment frame

<table>
<thead>
<tr>
<th>Design parameters</th>
<th>10% in 50 years hazard (2/3 MCE)</th>
<th>2% in 50 years hazard (MCE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_a$</td>
<td>0.3 g</td>
<td>0.45 g</td>
</tr>
<tr>
<td>$T$</td>
<td>3.36 s</td>
<td>3.36 s</td>
</tr>
<tr>
<td>$C_2$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Yield drift ratio $\theta_y$</td>
<td>0.5%</td>
<td>0.5%</td>
</tr>
<tr>
<td>Target drift ratio $\theta_u$</td>
<td>2%</td>
<td>3%</td>
</tr>
<tr>
<td>Inelastic drift ratio $\mu = \frac{\theta_u - \theta_y}{\theta_y}$</td>
<td>1.5%</td>
<td>2.5%</td>
</tr>
<tr>
<td>$R_\mu$</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.4375</td>
<td>0.3056</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.662</td>
<td>1.103</td>
</tr>
<tr>
<td>$V_y/W$</td>
<td>0.055</td>
<td>0.054</td>
</tr>
<tr>
<td>Design base shear $V_y$</td>
<td>255 kips (governs)</td>
<td>248 kips</td>
</tr>
</tbody>
</table>

*For one moment frame.

Figure 16. Floor plan of the 20-storey RC space moment frame building
It should be mentioned that the P-Delta term was not included in Equation (18). That is the way design of beams of the PBPD steel frame was performed. However, due to strength degradation at beam plastic hinges, it was found necessary to include P-Delta effect in the determination of required moment capacity of beams for the 20-storey RC frame. This was accomplished by adding ‘P-Delta lateral force’, $F_{i,PD}$, to the basic design force, $F_i$, in Equation (18). The force $F_{i,PD}$ can be taken equal to $P_i \theta_u$, where $P_i$ represents the tributary gravity load at floor level $i$, and $\theta_u$ represents the target design drift ratio that is assumed constant for design purposes. The values of $F_{i,PD}$ for the frame are shown in Table 5. Their influence on the total lateral design force can be clearly noticed as it has significant effect on the required frame strength.

The pushover curves for the two frames in Figure 17 show that even though the design base shear for the baseline frame is smaller than that of the PBPD frame, the ultimate strengths of the two are almost equal. That is mainly due to the fact that the design of the baseline frame was governed by drift that required major revision of the member sizes after having been designed for strength. That iteration step is not needed in the PBPD method. Calculated values of $R_{max}$ for the baseline and PBPD frames, according to the recommended equation in FEMA (2009b), are 5.3 and 10.8, respectively. This reflects much enhanced margin against dynamic instability (collapse) of the PBPD frame over that of the baseline frame. Figure 18 shows the deformed shape and location of plastic hinges of the two frames at 2.5% roof drift. Formation of plastic hinges in the columns and storey mechanism in the lower part of the baseline frame can be clearly noticed. In contrast, there are no unintended plastic hinges in the columns of the PBPD frame, resulting in a more favourable deformed shape and yield pattern as intended in the design process.

Figure 19 shows comparison of maximum interstorey drifts of the two frames as calculated by the energy spectrum method (discussed in Section 5) with those obtained from time-history analyses using...
Table 5. Lateral force distribution calculations for PBPD 20-storey RC moment frame

<table>
<thead>
<tr>
<th>Floor</th>
<th>$h_i$ (ft)</th>
<th>$w_i$ (kips)</th>
<th>$\beta_i$</th>
<th>$F_i$ (kips)</th>
<th>$F_{i,PD}$ (kips)</th>
<th>$F_i + F_{i,PD}$ (kips)</th>
<th>Storey shear $V_i^*$ (kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roof</td>
<td>262</td>
<td>231·8</td>
<td>1·00</td>
<td>63·6</td>
<td>4·6</td>
<td>68·2</td>
<td>68·2</td>
</tr>
<tr>
<td>20</td>
<td>249</td>
<td>231·8</td>
<td>1·48</td>
<td>30·6</td>
<td>4·6</td>
<td>35·2</td>
<td>103·4</td>
</tr>
<tr>
<td>19</td>
<td>236</td>
<td>231·8</td>
<td>1·85</td>
<td>23·6</td>
<td>4·6</td>
<td>28·2</td>
<td>131·7</td>
</tr>
<tr>
<td>18</td>
<td>223</td>
<td>231·8</td>
<td>2·16</td>
<td>19·6</td>
<td>4·6</td>
<td>24·2</td>
<td>155·9</td>
</tr>
<tr>
<td>17</td>
<td>210</td>
<td>231·8</td>
<td>2·42</td>
<td>16·8</td>
<td>4·6</td>
<td>21·4</td>
<td>177·3</td>
</tr>
<tr>
<td>16</td>
<td>197</td>
<td>231·8</td>
<td>2·65</td>
<td>14·7</td>
<td>4·6</td>
<td>19·3</td>
<td>196·6</td>
</tr>
<tr>
<td>15</td>
<td>184</td>
<td>231·8</td>
<td>2·86</td>
<td>12·9</td>
<td>4·6</td>
<td>17·6</td>
<td>214·1</td>
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<tr>
<td>14</td>
<td>171</td>
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<td>11·5</td>
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<tr>
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<td>14·8</td>
<td>245·1</td>
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<tr>
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<td>10</td>
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</tr>
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<td>67</td>
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<td>5</td>
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<td>231·8</td>
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</tr>
<tr>
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<td>41</td>
<td>231·8</td>
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<tr>
<td>2</td>
<td>15</td>
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<td>0·8</td>
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<td>5·4</td>
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<tr>
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<td></td>
<td>4636·0</td>
<td>62·11</td>
<td>255</td>
<td>92·7</td>
<td>347·3</td>
<td></td>
</tr>
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</table>

Figure 18. PH locations at 2·5% roof drift. (a) Code compliant frame; (b) PBPD frame; pushover analysis
Figure 19. Comparison of maximum interstorey drifts by the energy spectrum method and time-history analyses for (a) code compliant frame for 2/3 MCE; (b) code compliant frame for MCE; (c) PBPD frame for 2/3 MCE; and (d) PBPD frame for MCE hazard levels.
appropriately scaled ground motion records representative of 2/3 MCE and MCE hazard levels. The results show that the mean maximum interstorey drifts of the PBPD frame are well within the corresponding target values, i.e. 2% for 2/3 MCE and 3% for MCE. Moreover, the storey drifts of the PBPD frame are more evenly distributed over the height as compared with those of the baseline frame where undesirable ‘softness’ in the lower storeys is evident, which is caused mainly by plastic hinges in the columns. It is also worth noting that the interstorey drifts predicted by the energy spectrum method are in good agreement with those obtained from the dynamic analyses.

7. SUMMARY AND CONCLUSION

The PBPD method is a direct design method that uses pre-selected target drift and yield mechanisms as key performance objectives that determine the degree and distribution of expected structural damage. The design base shear for a specified hazard level is calculated by equating the work needed to push the structure monotonically up to the target drift to the energy required by an equivalent EP-SDOF to achieve the same state. Plastic design is performed to detail the frame members and connections in order to achieve the intended yield mechanism and behaviour. The method has been successfully applied to a number of commonly used steel framing systems. Development of the PBPD methodology for RC structures with degrading hysteretic behaviour is currently in progress. The examples of 20-storey steel and RC moment frames, as presented in this paper, showed that the method is especially advantageous for tall frames where the cumbersome and lengthy iterative process in current design practice can be completely eliminated, while leading to excellent performance as targeted.

The basic work-energy equation can also be used for seismic evaluation purposes where the goal is to determine expected displacement demand for a given structure and earthquake hazard. The results as presented in this paper showed excellent agreement with those obtained from more elaborate inelastic time-history analyses.

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