Note that I did not get to the terminology "Riemann sum" in class. However, the sums $L(n)$ and $R(n)$ are Riemann sums for $f$ over the interval $[a, b]$ as shown in class. Notice that later when we computed $L_p(n)$ and $R_p(n)$ that these are also Riemann sums for $f$ over the interval $[a, b]$. There are many such sums, depending on the partition $P$ we choose and the choices of the points $t_i^*$ in the subintervals.

I don’t like the way your text defined definite integral because we cannot compute the definite integral in the way that they indicate unless we know that the definite integral exists. That is, if we know the definite integral of $f$ over $[a, b]$ exists then we can compute it (or find it) using the limits in the book. However, just because one or the other of those limits exists this would not imply that the function was integrable. Your book skirts the issue by defining the definite integral for a continuous function—we can prove that if a function $f$ is continuous function on an interval $[a, b]$ then its definite integral over $[a, b]$ exists.

**Definition we were heading toward in class:**
Let $f$ be a function defined on a closed interval $[a, b]$. We say that a number $I$ is the **definite integral of $f$ over $[a, b]$** and that $I$ is the limit of the Riemann sums $\sum_{k=1}^{n} f(c_k) \Delta x_k$ if the following condition is satisfied:

Given any number $\varepsilon > 0$ there is a corresponding number $\delta > 0$ such that for every partition $P = \{x_0, x_1, \ldots, x_n\}$ of $[a, b]$ with $\|P\| < \delta$ and any choice of $c_k$ in $[x_{k-1}, x_k]$, we have $\left| \sum_{k=1}^{n} f(c_k) \Delta x_k - I \right| < \varepsilon$. 