Endogenous Inside-Debt Compensation*

Mingcherng Deng        Lin Nan        Xiaoyan Wen
Baruch College, CUNY   Purdue University Texas Christian University
December 15, 2016

Abstract

Previous studies and regulators usually regard inside-debt compensation as a device to mitigate the conflict of interests between shareholders and debtholders, and believe that firms need to be enforced to pay their executives inside-debt compensations to resolve this conflict. However, in practice it is very common that firms voluntarily offer inside-debt compensations to their executives, which cannot be explained by extant research.

In this paper we try to shed light on firms’ endogenous reasons of offering inside-debt compensation. We examine a setting in which a firm seeks external fund from a competitive debt market for its project, and offers a compensation package which may include both performance-based and inside-debt compensations to its manager. We consider three scenarios with different seniority orders in claiming against the liquidation value upon bankruptcy. We find that in all the three scenarios, the firm is self-motivated to offer inside-debt compensation to maximize its expected payoff. This is because, by choosing the “right mix” between the inside-debt and the performance-based compensations, a firm is able to calibrate its manager’s risk choice and achieve the first best to maximize its shareholders’ welfare. Without inside-debt compensation, the firm is unable to achieve the first best. In addition, the “right mix” between the performance-based compensation and the inside debt varies with the seniority orders of debts.

*We thank Divya Anantharaman, Viktoria Diser, Kyungha Lee, Haijin Lin, Joyce Tian, Bharat Sarath, Phillip Stocken, James R. Thompson, seminar participants at University of Waterloo, Rutgers University, participants of the 2016 MAS Midyear Conference at Dallas, and participants of the 2016 AAA Annual Meeting at NYC for helpful comments.
1 Introduction

Performance-based compensation has been in the spotlight in the research of managerial compensation for years, yet only recently has debt-like compensation begun to attract more attention from both researchers and regulators. In practice, managers receive not only performance-based compensation, such as bonuses and stock options, but also debt-like compensation, such as pensions and deferred compensation (also known as inside debt compensation). It is important to understand the role of inside-debt compensation at least for two reasons. First, executive pensions and other debt-like compensation constitutes significant portions of executives’ compensation (Bebchuk and Jackson, 2005; Sundaram and Yermack, 2007; Gerakos, 2010; Wei and Yermack, 2011). Second, overall compensation of top managers are very sensitive to a change in the actuarial value of their pension compensation. Thus, the magnitude of inside debt compensation significantly affects the incentives of top managers’ risk taking decisions (Sundaram and Yermack 2007).

Conventional wisdom usually regard inside-debt compensation as a device to mitigate the conflict of interests between shareholders and external debtholders. The argument is that if an executive is only compensated with performance-based compensation, the executive would take excessive risk to increase shareholders’ welfare at debtholders’ expenses. For that reason, critiques of compensation practices often attributes the financial crisis around 2008 partially to performance-based compensation. Therefore, to protect the interest of external debtholders, regulatory intervention might be necessary by forcing firms to pay their executives not only performance-based compensation but also inside-debt compensation (Anantharaman et al., 2014; Baily et al., 2013; Bolton et al., 2015; Edmans and Liu, 2011; French et al., 2010). To curb managers’ imprudent risk-taking behavior, Section 956 of the Dodd-Frank Act requires U.S. financial regulators to issue rules to defer a certain portion of incentive-based compensation awarded to senior executive officers or significant risk-takers for a specified period of time.\footnote{U.S. regulators jointly proposed in 2011 and then re-proposed in 2016 rules for Section 956. This regulation requires that all large financial institutions ($50 billion or more total consolidated assets) defer a substantial portion of incentive-based compensation of senior executive officers or significant risk-takers.}

This line of arguments may not present a whole picture of the economic benefits of inside-debt compensation. First, many firms voluntarily offer their managers inside-debt compensation even in the absence of compensation regulations such as Section 956 of the Dodd-Frank Act. Sundaram and
Yermack (2007) show that the percentage of inside debt exceeds the percent of equity for thirteen percent of CEOs. Why would shareholders voluntarily provide CEOs inside-debt compensation? The popularity of inside-debt compensation indicates that firms must have some endogenous benefits to offer inside-debt compensation.

Second, in the presence of both inside debts and external debts, it is important to consider the relative seniority order of those debts, because intuitively whether external debts or inside debt compensation is more senior will impact top managers’ risk incentives. One may believe that external debts are always more senior than inside debt since the CEO’s inside debt compensation usually has longer duration than external debts. But Calcagno and Renneboog (2007) examine bankruptcy regulations in countries such as US, UK and Germany and show the cases in which managerial remuneration claims are senior to external debts. Thus, given the mixed practices, it is important to understand the role of relative seniority orders in inside debt compensations. Perhaps because this unique feature is irrelevant in performance-based equity compensation, the extant literature and regulations have been silent on the economic effects of seniority orders.

Motivated by these two observations, in this paper we examine the role of inside-debt compensation in a setting in which a firm seeks external fund from a competitive debt market to finance its project, and offers a compensation package, which may include both performance-based and inside-debt compensation, to its manager. In particular, we consider three scenarios with different seniority orders in claiming against the liquidation value upon bankruptcy: 1) the equity-seniority scenario in which the manager and the creditor have equal seniority, 2) the creditor-seniority scenario in which the creditor has the seniority to claim the liquidation value, and 3) the manager-seniority scenario in which the manager has the seniority to claim the liquidation value before the creditor.

We find that in all the three scenarios, a firm is self-motivated to offer inside-debt compensation to maximize its shareholders’ expected payoff, because the firm is able to calibrate the manager’s risk choice through adjusting the mix of inside-debt and performance-based compensation. By choosing the “right mix” between the performance-based and the inside-debt compensation, the firm is able to induce the manager to choose the first-best risk level; the performance-based compensation encourages the manager to take more risk whereas the inside debt discourages the manager from taking risk. Because the manager’s and the firm’s interests are not fully aligned due to the frictions of debt financing, without the option to offer inside debt, the firm is unable to induce the manager
to choose the first-best risk level. In addition, the “right mix” between the performance-based compensation and the inside debt varies with the seniority orders of debts; that is, different scenarios have different “right mix” ratios.

Moreover, we compare the equilibrium inside-debt compensation across the three scenarios with different seniority orders. Our analysis shows that, when the borrowing amount is small, the inside-debt compensation in the creditor-seniority scenario is the lowest and the inside-debt compensation in the manager-seniority scenario is the highest. This is because when the creditor has the seniority, the manager tends to take less risk as he receives no payment upon failure. To motivate the manager to take the higher first-best risk level, the firm offers higher performance-based compensation and lower inside-debt compensation. In contrast, when the manager has the seniority, the manager tends to take more risk because he is well protected upon failure. To motivate the manger to choose the first-best risk level, the firm offers lower performance-based compensation and higher inside-debt compensation. As the borrowing amount increases, our analysis shows that the inside-debt compensation in all the three scenarios increase, but the inside-debt compensation in the manager-seniority scenario increases at the slowest pace; when the borrowing amount is sufficiently large, the inside-debt compensation in the manager-seniority scenario becomes the smallest among the three scenarios. The reason is that when the borrowing amount is large, most of the project return will be paid to the creditor as repayment, and the profit left for the firm will be small; therefore, the manager’s performance-based compensation is very limited as the performance-based compensation is proportional to the firm’s profit after debt liabilities. As a result, the firm must increase inside-debt compensation to satisfy the manager’s reservation utility in all the three scenarios. However, in the manager-seniority scenario the firm does not need to increase the inside-debt compensation so much as in the other two scenarios, because the manager is better protected when he has the seniority.

We also analyze how the creditor’s requested repayment changes with the liquidation value upon bankruptcy. Conventional wisdom may believe that when the liquidation value is higher, the creditor’s downside risk would be better protected and thus the requested repayment must decrease. However, our analysis shows that the requested repayment may increase in the liquidation value. Specifically, there are two countervailing effects of the liquidation value. On one hand, a higher liquidation value protects the creditor’s downside risk upon project failure, and thus it induces a
lower debt repayment. We call it the protection-upon-failure effect. On the other hand, when the liquidation value is high, the manager has a stronger incentive to choose higher risk, which induces a higher debt repayment. We call this effect the risk-taking effect. We find that in the equal-seniority scenario and the creditor-seniority scenario, the requested repayment may increase or decrease with the liquidation value, and which effect dominates depends on factors such as borrowing amounts. In the manager-seniority scenario, the risk-taking effect always dominates, and thus the requested repayment always increases in the liquidation value.

We further discuss the case where the manager is risk averse. The firm faces an additional economic trade-off between the benefit of inducing a risk-averse manager to take a higher risk level and the cost of paying higher risk premium to the manager. In equilibrium, the firm is better off to induce the manager to pick a risk level lower than the optimal level to save the risk premium. The firm’s ex ante payoff in the risk averse setting is lower, because (i) the project risk level is lower than the optimal level, which leads to lower the expected project cash flow and (ii) the firm has to compensate the manager for his risk premium.

The rest of the paper is organized as follows: Section 2 discusses related literature. In Section 3 we introduce the model and analyze the first-best benchmark. In Section 4 we examine the three scenarios with different seniority orders, and in Section 5 we compare the equilibrium choices as well as some interesting comparative statics among the three scenarios. In Section 6, we discuss the case where the manager is risk averse and provide empirical implications. Section 7 concludes the paper.

2 Related Literature

Most extant studies on inside-debt compensation focus on how to design inside debt to resolve the conflict of interest between shareholders and debtholders. For example, Baily et al. (2013) point out that stockholders prefer firms to take more risk than debtholders do, and government bailouts further exacerbate financial institutions’ excessive risk taking. They propose that financial institutions should be forced to withhold one fifth of each senior manager’s total annual compensation for five years in order to mitigate the excessive risk taking problem. Edmans and Liu (2011) examine a setting with exogenous leverage. They model the social welfare as the sum of both
shareholders’ and debtholders’ welfare and they argue that inside debt is a superior solution to the “asset-substitution” problem than the solvency-contingent bonuses and salaries proposed by prior literature, because its payoff depends not only on the incidence of bankruptcy but also firm value in bankruptcy. Anantharaman, Fang and Gong (2014) examine the average effect of executive debt-like compensation on loan contracting terms. Based on empirical evidence, they show that debt-like compensation is only effective at resolving stockholder-debtholder conflicts when its payoffs are truly debt-like (that is, its payoffs are truly exposed to risk of loss in insolvency). These papers are mostly concerned about the effectiveness of inside debt to resolve the conflict of interest between shareholders and debtholders, and they usually believe that regulatory measures are needed to enforce the use of inside-debt compensation. In contrast, our study shows that, to maximize shareholders’ welfare, firms actually prefer to offer inside-debt compensation to their executives. This is because with inside debt firms are able to achieve first best which maximizes their shareholders’ payoff.

There are a few studies on the effects of inside-debt compensation other than resolving the shareholders-debtholders conflicts. For example, Eaton and Rosen (1983) study delayed compensation and the structure of executive remuneration, and they show that delayed compensation helps to align managers’ incentive with shareholders’ long-run interest. Our paper differs from their study as in our model inside-debt compensation is not a means to resolve managers’ short-termism but a measure to induce optimal risk choice to maximize shareholders’ welfare. Allen and Thompson (2016) examine a contracting problem with risk-averse managers. They show that there is a trade-off when a firm offers debt-like ex-post fixed payment to its manager: On one hand, the firm’s compensation cost is lower by offering a fixed payment instead of a variable payment because the manager is risk-averse; on the other hand the firm has to reduce other debts to avoid bankruptcy. In contrast, in our model we focus on the role of inside debt compensation to help a firm calibrate its manager’s risk choice.

The extant literature has been focused on studying the effects of stock compensation on alleviating the agency conflicts between shareholders and managers. But very few analyze the role of inside-debt compensation. Arya and Mittendorf (2005) find that option-based compensation can help to improve the matching between managerial ability and compensation. Arya, Fellingham and Glover (1997) show that implicit incentive contracts may be optimal to motivate team members in
a setting in which only team performance is tracked. Wagenhofer (2003) shows that compared with cash flows, accruals may be a better performance measure in compensation contracting. Bertomeu (2015) studies whether incentive contracts should expose managers to market shocks, and he finds that with an agent of decreasing absolute risk aversion, a principal should increase the performance pay following a favorable market shock. Chen, Hemmer and Zhang (2011) show that loose monitoring can be beneficial because it may achieve separation among different types of firms, such that firms with low potential do not have incentives to imitate contracts offered by high potential firms. Christensen, Sabac and Tian (2010) derive sufficient conditions for ranking performance measures in multi-task agency settings. Baldenius, Glover and Xue (2015) study the trade-off between verifiable team performance and non-verifiable individual measures. They show that under productive complements, an unconditional bonus pool (pay without performance) can be less costly than one conditioned on the verifiable team measure. Demski and Sappington (1984) examine a contracting problem in a setting in which a principal contracts with multiple agents whose productivities are correlated, and they find that each agent’s private information regarding his own productivity may not be able to provide the agent with rent. Our study deviates from this line of literature because we focus on the role of inside-debt compensation instead of performance-based compensation.

3 The Model

We consider a model in which a firm seeks a fund of $I - A$ from a representative risk-neutral creditor to implement a project, where $I$ is the needed investment and $A$ is the firm’s own assets, and $I$ and $A$ are common knowledge. At date 1, the firm offers a compensation package to a manager, who has a reservation utility $U$. The compensation package is publicly observable. For convenience, we hereafter refer to the manager as “he” and refer to the creditor as “she.” At date 2, the representative creditor decides on whether to fund the firm; if she decides to fund the project, she provides $I - A$ and asks for a repayment $F$. We assume that there is a competitive debt market and therefore the representative creditor chooses the requested repayment to break even.

At date 3, the manager privately chooses the risk level of the firm’s project, which is reflected in the probability of success, $p \in (0, 1]$. With probability $p$, the project will be successful and generate
a cash flow

\[ R(p) = (-\ln p) + r_f, \]

where \( r_f > 0 \). The cash flow upon project success \( R(p) \) decreases in \( p \); that is, the higher the chance to succeed, the lower the expected return upon project success.\(^2\) With probability \( 1 - p \), the project fails and the firm is bankrupt, and the project generates a liquidation value \( r_l < I - A \). The cash flows generated by the project are summarized as follows:

\[
\begin{cases}
R(p), & \text{with a probability } p; \\
r_l, & \text{with a probability } 1 - p.
\end{cases}
\]

We assume \( r_f > r_l > 0 \) so that the liquidation value is lower than the minimal cash flow when the project is successful.

We use the probability of success \( p \) to capture the trade-off between the project risk and the project cash flow. If \( p \) is low, the project has a high likelihood of failure, but the cash flow upon project success is high. That is, the cash flow distribution’s spread is larger for a higher-risk project (i.e., the difference between \( R(p) \) and \( r_l \) is large for a low \( p \)). This suggests that when the project is of high risk, the cash flow volatility is also high. In contrast, if \( p \) is high, the project has a low likelihood of failure and is less risky, but the cash flow upon project success is also low. In other words, the cash flow volatility is low when the project is of low risk. In the extreme case where \( p \) is 1, the project always succeeds and never fails. In this case, the project cash flow always equals to \( r_f \) and the cash flow volatility is zero. For example, depositing money into a bank account could be regarded as a risk-free investment because the probability of success is almost one and there is no cash flow volatility; on the other side, investing money in a venture capital fund could be regarded as a highly-risky investment because the probability of success is relatively low, but the cash return upon project success is way higher than that from a bank deposit.

As we mentioned in the introduction, in the real world managers not only receive compensation based on performance such as bonuses, but also receive inside-debt compensation such as

\(^2\)The main results of our model do not depend on the function form of \( R(p) \) as long as \( R(p) \) decreases in \( p \). That is, regardless of the function form of \( R(p) \), in equilibrium the firm always offers positive inside-debt compensations and the firm achieves the first best by choosing the “right mix” between the performance-based and inside-debt compensations. Detailed analysis with a general function form of \( R(p) \) is included in Appendix I.
pensions and other deferred compensation. To reflect this fact in our model, we assume that the compensation package offered to the manager may have two components. One component of the compensation is a compensation directly based on the performance; specifically, the manager obtains $\alpha$ portion of the firm’s equity value as his compensation. The other component is inside-debt compensation; the manager gets $B$ as his inside-debt compensation, which is a liability of the firm. For our convenience, we call the first component the **performance-based compensation**, and call the second component the **inside-debt compensation**. Notice that it may not be necessary to have both components in the compensation package and we allow either component to be zero. The compensation package $\{\alpha, B\}$ is publicly observable and verifiable.

Also note that the inside-debt component is very different from fixed payments studied in many prior papers. A fixed payment is a guaranteed payment to the manager, whereas the inside-debt compensation is not. The inside-debt compensation is subject to the project outcome as well as the relative seniority order between inside and external debts. If the project fails, the manager may not be able to receive the full amount of his inside-debt compensation, or even may not receive any inside-debt compensation, depending on the relative seniority order.

If the representative creditor decides to fund the firm’s project, she provides $I - A$ and asks for a repayment $F$ based on the observed compensation package $\{\alpha, B\}$ and her conjecture about the manager’s risk-selection decision. As the debt market is competitive, the creditor determines $F$ to break even; that is, the expected repayment from the firm equals the expected cost of lending.

---

3In the current model we do not consider a guaranteed fixed payment. However, it can be easily included in our analysis. If the manager also receives a guaranteed fixed payment before the outcome is realized in his compensation package, the fixed payment can be applied against the manager’s reservation utility. All our results would qualitatively remain the same as if the manager’s reservation utility is lowered. In equilibrium we still achieve the first-best risk level and the inside-debt compensation is still positive.
The time line is illustrated below.

<table>
<thead>
<tr>
<th>Date 1</th>
<th>Date 2</th>
<th>Date 3</th>
<th>Date 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>The firm offers a compensation package, ( {\alpha, B} ), to the manager.</td>
<td>The creditor makes the funding decision, and if she decides to fund, the project’s outcome is realized. The firm repays the creditor.</td>
<td>The manager chooses the project’s risk level. and compensates the manager.</td>
<td>The project’s outcome is realized. The firm repays the creditor.</td>
</tr>
</tbody>
</table>

Time line.

Inside debts are a part of a firm’s liabilities. A manager who holds inside-debt compensation is also a debtholder of the firm. Because both the internal manager and the external creditor hold debts of the firm, it is important to consider the relative seniority order of those debts. Upon bankruptcy, different relative seniority orders influence the allocation of the liquidation value \( r_l \) between the manager and the creditor, and thus affect the manager and the creditor’s ex-ante decisions. We consider three scenarios of different seniority orders between the inside debt of the manager and external debt of the creditor: 1) **equal-seniority scenario** in which the inside and the external debts have the same seniority, 2) **creditor-seniority scenario** in which the creditor gets her claim against the liquidation value first, and 3) **manager-seniority scenario** in which the manager has the seniority to claim against the liquidation value before the external creditor.

We now define the equilibrium in our model:

**Definition 1** *The equilibrium consists of the firm’s compensation package \( \{\alpha^*, B^*\} \), the creditor’s repayment decision \( F^*(\cdot) \), and the manager’s decision \( p^*(\cdot) \), such that:*

- *the firm chooses the compensation package \( \{\alpha^*, B^*\} \) to maximize its expected payoff \( \tilde{p} (1 - \alpha) [R(\tilde{p}) - F - B] - A \), given the firm’s conjecture of the manager’s decision \( \tilde{p}(\cdot) \), subject to a constraint that the manager should get an expected payoff that is not lower than \( U; \)*
• the creditor chooses $F^\ast$ to break even, given the observable compensation package and the creditor’s conjecture of the manager’s decision $\hat{p}(.);$ 

• the manager chooses $p^\ast$ to maximize his expected payoff;

• the firm’s and the creditor’s conjectures are correct (i.e., $\hat{p}(.)=p^\ast(.)$).

In the above definition, both the manager’s payoff and the creditor’s payoff are affected by the seniority order because the seniority order determines how the liquidation value $r_l$ is split between the manager and the creditor upon bankruptcy. We will analyze the equilibrium in each seniority scenario respectively in the next section.

3.1 First-Best Benchmark

Before analyzing the three scenarios, we first analyze the first-best benchmark. In the first-best case, there is no need of debt financing (the firm has sufficient fund to undertake its project), and the manager’s risk choice is observable and verifiable (the firm is able to enforce the manager’s risk choice). The first-best risk level is chosen to maximize the firm’s expected payoff in this benchmark, which is denoted by $\Pi^o$,

\[ \Pi^o \equiv p \cdot R(p) + (1-p)r_l - I - U \]

where the first two terms represents the expected project cash flow, $I$ is the total investment, and $U$ is the manager’s reservation utility. The first-order condition to solve for the first-best risk level is

\[ \frac{\partial \Pi^o}{\partial p} = R(p) - 1 - r_l = 0, \]  

which gives $(-\log p) + r_f - 1 - r_l = 0$. Therefore, the first-best risk level to maximize the project’s NPV, denoted by $p^o$, is

\[ R(p^o) = r_l + 1 \text{ or } p^o = \exp[r_f - 1 - r_l]. \]

The first-best risk level is lower (i.e., $p^o$ is higher) when the risk-free return $r_f$ is higher or when the liquidation value $r_l$ is lower (i.e., $\frac{\partial p^o}{\partial r_f} > 0$, $\frac{\partial p^o}{\partial r_l} < 0$). Intuitively, when the risk-free return $r_f$ is
higher, the incremental benefit of taking more risk becomes lower, and thus the first-best risk level gets lower. Similarly, when the liquidation value $r_l$ is lower, the incremental loss from taking more risk is higher, and thus the first-best risk level becomes lower.

4 Three Scenarios with Different Seniority Orders

We now analyze our main setting. We consider different scenarios with different seniority orders in claiming against the liquidation value upon bankruptcy. The seniority order affects both the manager’s and the creditor’s decisions because it determines how the liquidation value $r_l$ is split between the manager and the creditor upon bankruptcy. We analyze three possible scenarios: 1) the equity-seniority scenario in which the manager and the creditor have equal seniority, 2) the creditor-seniority scenario in which the creditor has the seniority to claim the liquidation value, and 3) the manager-seniority scenario in which the manager has the seniority to get his payment out of the liquidation value before the creditor. Our goal is to examine how seniority orders influence the firm’s and the creditor’s strategies, and how seniority affects the endogenous inside-debt compensation.

To avoid uninteresting or trivial cases, we employ two assumptions in the following analysis. First, we assume $r_f < 1 + r_l$, which implies that the risk-free return $r_f$ is sufficiently low so that the first-best choice is not to choose risk free (i.e., $p^0 \neq 1$). Second, we assume $p^0 + r_l - \bar{U} > I$, so that the firm’s ex-ante expected payoff with risk level $p^0$ is positive (i.e., $\Pi^0 = p^0 + r_l - I - \bar{U} > 0$); otherwise the firm is not willing to initiate the project at the first place. This also implies that $p^0 > \bar{U}$ because the liquidation value is lower than the investment cost ($r_l < I$).

4.1 Equal-seniority Scenario

In the equal-seniority scenario, the manager and the creditor have equal seniority in claiming against the liquidation value when the project fails. The liquidation value $r_l$ is allocated between the manager and the creditor in proportion of the face value of inside debt ($B$) and external debt ($F$). The proportion is determined by the relative weights between the inside debt and the external debt. That is, upon project failure and thus bankruptcy, the creditor receives a proportion of the liquidation value $r_l \frac{F}{F+B}$ for the external debt. The manager receives $r_l \frac{B}{F+B}$ for the inside debt, and receives no performance-based compensation because the firm’s equity value is zero at bankruptcy.
Upon project success, both inside and external debts will be settled with full repayments, and the firm’s equity value after paying off liabilities is $R(p) - F - B$. The manager’s expected payoff in this scenario is

$$U_M = \alpha \cdot p \cdot [R(p) - F - B] + p \cdot B + (1 - p)r_l \frac{B}{F + B},$$

where the first term represents the expected performance-based compensation, the second term reflects the expected inside-debt compensation when the project is successful, and the last term is the proportion of the liquidation value the manager receives upon bankruptcy. In addition, to ensure the manager’s participation, the firm should offer a compensation package that makes the manager’s expected payoff $U_M$ to be at least $\overline{U}$.

The compensation package the firm offers will affect the manager’s risk choice. To see how the performance-based and the inside-debt compensation influence the manager’s choice, we look at the first-order condition for the manager to choose the risk level:

$$\frac{\partial U_M}{\partial p} = \alpha [R(p) - F - B - 1] + B - r_l \frac{B}{F + B} = 0,$$

which can be rewritten as

$$\ln p = [F + B - r_l] \left[ \frac{B}{F + B} \cdot \frac{1}{\alpha} - 1 \right] + r_f - 1 - r_l. \quad (2)$$

The manager trades off between the benefit of increasing the performance-based compensation and receiving the inside debt upon success, and the cost of lower compensation from the liquidation value upon bankruptcy. From Equation (2), it is easy to verify that the manager will choose higher risk (i.e., lower $p$) when the performance-based compensation $\alpha$ is higher, but will choose lower risk when the inside-debt compensation $B$ is higher. Intuitively, if the manager’s performance-based compensation weighs more, the manager cares more about the upside benefit from the project than the downside risk, and therefore prefers to take more risk. On the other hand, if the manager’s inside-debt compensation weighs more, the manager is more aligned with the creditor, concerned more about the downside risk and thus prefers less risk.
The creditor’s expected payoff in this scenario is

\[ U_C = \hat{p} \cdot F + (1 - \hat{p}) \cdot r_l \frac{F}{F + B} - (I - A), \]

and the creditor’s expected payoff should be zero in equilibrium because the debt market is competitive. Notice that given the observable compensation package, the creditor can accurately conjecture the manager’s risk choice.

Denoting the equilibrium choices in the equal-seniority scenario to be \( \alpha^*_e, B^*_e, F^*_e \) and \( p^*_e \), we have the following results:

**Proposition 1** In the equal-seniority scenario, the firm offers \( \alpha^*_e = \frac{B^*_e}{B^*_e + F^*_e} = \frac{\overline{r}_l}{\overline{r}_l + r_l} \) and \( B^*_e = \frac{\overline{U}(I - A)}{\overline{r}_l \overline{p}^*_e + r_l - \overline{U}} - \frac{\overline{U} \overline{r}_l (1 - p^*_e)}{\overline{p}(p^*_e + r_l)} \), the creditor requests a repayment \( F^*_e = \frac{I - A - (1 - p^*_e) r_l}{p^*_e} + \frac{\overline{U} r_l (1 - p^*_e)}{p^*_e (p^*_e + r_l)} \), and the manager chooses \( p^*_e = p^o \).

Our analysis shows that the first-best risk level is achieved in equilibrium in the equal-seniority scenario \( (p^*_e = p^o) \). To understand this result, notice that the first best is achieved when the manager’s expected payoff is fully aligned with the firm’s (or, the shareholders’) first-best expected payoff. The firm’s expected payoff in the first best is

\[ \Pi^o = p \cdot R(p) + (1 - p) r_l - I - \overline{U}, \quad (3) \]

while the manager’s expected payoff in the equal-seniority case can be rewritten into

\[ U_M = \alpha \cdot p \cdot R(p) + (1 - p) r_l \frac{B}{F + B} - p(F + B) \cdot \left( \alpha - \frac{B}{F + B} \right). \quad (4) \]

It is easy to verify that, to make these two expected payoffs congruent, we need

\[ \alpha = \frac{B}{F + B}. \quad (5) \]

Notice that \( \alpha \) represents the proportion of equity value that is assigned to the manager as his performance-based compensation, and \( \frac{B}{F + B} \) represents the proportion of inside debt in total debt. \( \frac{B}{F + B} \) is also the manager’s share of the liquidation value upon bankruptcy in this scenario.
When the firm offers a compensation package that satisfies $\alpha = \frac{B}{F + B}$, the manager will choose the first-best $p^0$. That is, by choosing the “right mix” between the performance-based compensation and the inside-debt compensation, the firm is able to induce the manager to pick the first-best risk level to maximize the firm’s expected payoff. Notice that as long as the compensation package satisfies $\alpha = \frac{B}{F + B}$, the manager will choose the first-best risk level, and thus there could be more than one pair of $\alpha$ and $B$ that can induce $p^0$. However, in equilibrium the firm will choose the specific pair of $\alpha$ and $B$ which makes the manager’s expected payoff exactly his reservation utility $U$. One can easily verify that this specific pair is the optimal compensation package to maximize the firm’s ex-ante expected payoff, as there is no friction from debt financing (the creditor breaks even in a competitive debt market) and the manager’s expected payoff is exactly his reservation utility. In other words, in equilibrium the firm achieves the first best and its expected payoff is exactly the first-best expected payoff $\Pi^0$.

**Corollary 1** In the equal-seniority scenario,

- the firm offers a compensation package that satisfies $\alpha = \frac{B}{F + B}$ to induce the manager to choose $p^0$;
- in equilibrium the inside-debt compensation in the compensation package is positive ($B_e^* > 0$).

Proposition 1 shows that the optimal inside-debt compensation that the firm offers to the manager is always positive (i.e., $B_e^* > 0$). This result may be surprising. Many studies in finance literature believe that regulators must enforce inside debt in order to improve social efficiency by solving the conflict between shareholders and debtholders (Edmans and Liu, 2011; Baily et al., 2013). In contrast, Proposition 1 indicates that a firm (or its shareholders) is self-motivated to offer inside-debt compensation, because the firm can calibrate its manager’s risk choice with both inside-debt compensation and performance-based compensation. Without inside-debt compensation, the firm would be unable to achieve the first best.

To further illustrate the importance of the inside-debt compensation, suppose that the firm cannot offer inside debt (i.e., suppose we force $B$ to be zero). Then the firm’s expected payoff

---

4 Notice that for any given compensation package $\alpha$ and $B$, the creditor will choose a corresponding $F$ to break even, and the firm is able to anticipate the creditor’s choice of $F$ when it chooses $\alpha$ and $B$.

5 Further details about the analysis of the case of enforcing $B = 0$ is included in Appendix II.
becomes \((1 - \alpha) \cdot p \cdot [R(p) - F(\alpha)]\) and the manager’s expected payoff becomes \(U_M = \alpha \cdot p \cdot (R(p) - F)\).

It is important to notice that the manager’s expected payoff is not aligned with the firm’s (or the shareholders’) expected payoff, although they “seem” to be aligned. The difference between these two payoffs is that the manager makes his risk decision ex post upon a given debt term, while the shareholders make the decision on \(\alpha\) ex ante, considering the creditor’s corresponding requested repayment \(F(\alpha)\). Without inside-debt compensation, the manager only cares about the upside potential and thus will choose a risk level that is higher than the first-best risk level, which does not maximize the shareholders ex ante expected payoff. Without the option to offer inside-debt compensation the shareholders have no measure to influence the manager’s risk choice. Only when the firm offers both performance-based and inside-debt compensation can the firm induce the manager to choose \(p^o\) to achieve the first best.

Although the debt contracting brings frictions and causes the conflict of interests between the manager and the shareholders, debt contracting in our model does not bring any inefficiency: As the compensation package is observable, the creditor is able to correctly conjecture the manager’s risk decision, and the creditor requests for a repayment to break even because the debt market is competitive.

Notice that the “right mix” does not depend on the function form of \(R(p)\). The “right mix” achieves congruence between the manager’s expected payoff \(U_M\) in Equation (4) and the firm’s first-best expected payoff \(\Pi^o\) in Equation (3), thereby inducing the first-best risk level in equilibrium. Intuitively, for any \(R(p)\), the firm is always able to calibrate the manager’s risk choice to the corresponding first-best \(p^o\) by choosing the right \(\alpha\) and \(B\) (the creditor’s choice of \(F\) will change accordingly as she is able to conjecture the risk choice accurately upon observing \(\alpha\) and \(B\)). As the “right mix” does not depend on the function form of \(R(p)\), the result that in equilibrium the firm always offers a positive inside-debt compensation does not depend on \(R(p)\) either. We include a detailed analysis in Appendix I, showing that with a general function form of \(R(p)\) our main results still go through.

### 4.2 Creditor-seniority Scenario

We now consider the case in which the creditor has the seniority. In this case, upon bankruptcy, the creditor obtains the full amount of the liquidation value \(r_l\) because the liquidation value is smaller
than the face value of the external debt (i.e., $r_l < I - A < F$). In contrast, the manager receives nothing upon bankruptcy. As a result, due to the creditor’s seniority, the inside-debt compensation does not provide any down-side protection to the manager upon project failure. The manager’s expected payoff, therefore, is

$$U_M = \alpha \cdot p \cdot (R(p) - F - B) + p \cdot B.$$ 

The liquidation value does not directly affect the manager’s payoff. When the project is successful, the manager will receive the performance-based compensation $\alpha \cdot (R(p) - F - B)$ and the inside debt $B$. When the project fails, however, the manager will not receive any portion of the liquidation value nor any performance-based compensation.

The creditor’s expected payoff in this scenario is

$$U_C = \hat{p} \cdot F + (1 - \hat{p}) \cdot r_l - (I - A),$$

which should be zero because the debt market is perfectly competitive.

Following a similar analysis as in the equal-seniority scenario, we solve for the equilibrium in this creditor-seniority scenario. Defining $\alpha^*_c, B^*_c, F^*_c$ and $p^*_c$ to be the players’ choices in the equilibrium, we have the following results:

**Proposition 2** In the creditor-seniority scenario, the firm offers $\alpha^*_c = \frac{B^*_c}{r_c + B^*_c - r_l} = \frac{\hat{p}}{p_c}$ and $B^*_c = \frac{F^*_c}{p_c} \frac{(I - A - r_l)}{p_c(p^*_c - \hat{p})}$, the creditor requests $F^*_c = \frac{I - A - (1 - p^*_c) r_l}{p_c}$, and the manager chooses $p^*_c = p^0$.

Proposition 2 shows that in the creditor-seniority scenario, again, the first-best risk level is achieved. By using both the performance-based and the inside-debt compensation, the firm calibrates the manager’s risk choice to be the first-best $p^0$. However, the “right mix” between $\alpha$ and $B$ to induce the first-best risk level is different from that in the equal-seniority scenario. To induce the manager to choose $p^0$, here the firm offers a compensation package that satisfies $\alpha = \frac{B}{r + B - r_l}$. Compared with the manager’s payoff in the equal-seniority case in (4), here the manager cannot obtain any of the liquidation value upon project failure, as the creditor has the seniority and gets all the liquidation value. As a consequence, the manager tends to take less risk because he receives no payment upon failure and thus the inside debt does not protect him upon bankruptcy.
To induce the manager to take a higher first-best risk level, the firm must offer a higher portion of equity as the performance-based compensation. Therefore, the proportion of equity as performance-based compensation is higher than the inside-debt proportion in the equal-seniority scenario \( \alpha = \frac{B}{F+BR-BF} > \frac{B}{F+B} \).

In equilibrium, the compensation package will also make the manager’s payoff exactly his reservation utility. This compensation package is the optimal one to maximize the firm’s expected payoff, because (i) it induces the first-best risk choice, (ii) the manager’s expected payoff is his reservation utility, and (iii) the creditor breaks even with accurate conjecture of the manager’s risk choice. Same as in the equal-seniority scenario, with the creditor seniority the firm also achieves the first best and in equilibrium the firm’s expected payoff is exactly \( \Pi^o \).

Again, in this scenario the optimal inside-debt compensation \( B^*_c \) is positive; without the inside-debt compensation, the firm would be unable to calibrate the manager’s risk choice to the first-best level \( p^o \) and the firm’s ex-ante expected payoff would be lower.

**Corollary 2** *In the creditor-seniority scenario,*

- the firm offers a compensation package that satisfies \( \alpha = \frac{B}{F+BR-BF} \) to induce the manager to choose \( p^o \);
- in equilibrium the inside-debt compensation in the compensation package is positive \( B^*_c > 0 \).

**4.3 Manager-seniority Scenario**

Managers’ inside-debt compensation is not always junior to external debts. Calcagno and Renneboog (2007) examine bankruptcy regulations in countries such as US, UK and Germany and show that there are cases in which managerial remuneration claims are senior to external debts. If the manager’s inside-debt compensation has higher seniority than external debts in claiming against the liquidation value, upon bankruptcy, the manager’s inside debt is settled before any repayment to the creditor. We need to consider two cases in this scenario: when the inside debt is larger than the liquidation value \( (B > r_l) \), all liquidation value is paid to the manager and the creditor receives nothing upon bankruptcy; otherwise \( (B \leq r_l) \), the inside debt \( B \) is paid to the manager and the remaining balance \( r_l - B \) is paid to the creditor upon bankruptcy. In this scenario, the manager’s
expected payoff is

\[ U_M = \alpha \cdot p \cdot (R(p) - F - B) + p \cdot B + (1 - p) \min\{r_l, B\}, \]

where the first two terms represent the manager’s payoff when the project is successful, and the last term is the payoff when the project fails. The first-order condition shows that to induce the first-best risk choice, the compensation package must satisfy

\[ \alpha = \frac{B - \min\{r_l, B\}}{F + B - r_l}. \]

The creditor’s expected payoff is

\[ U_C = \hat{p} \cdot F + (1 - \hat{p}) \cdot \max\{r_l - B, 0\} - (I - A), \]

which should be zero in equilibrium since the debt market is perfectly competitive.

As we will soon show, depending on the relationship between \( \overline{U} \) and \( r_l \), we have two cases with different equilibrium choices. We use \( \{\alpha_{m1}^*, B_{m1}^*, F_{m1}^*, p_{m1}^*\} \) to denote the equilibrium choices in the manager-seniority scenario when \( \overline{U} > r_l \), and use \( \{\alpha_{m2}^*, B_{m2}^*, F_{m2}^*, p_{m2}^*\} \) to denote the equilibrium choices when \( \overline{U} \leq r_l \), respectively.

The case of \( \overline{U} > r_l \): In the case of \( \overline{U} > r_l \), the manager’s reservation utility is larger than the liquidation value. Because the manager’s expected payoff cannot be lower than the reservation utility, we conjecture that the inside-debt compensation \( B \) is higher than the liquidation value (that is, \( \min\{r_l, B\} = r_l \)). We will later verify that this conjecture is indeed true in equilibrium. Given the conjecture that \( \min\{r_l, B\} = r_l \), we solve the equilibrium and have

\[ \alpha_{m1}^* = \frac{B_{m1}^* - r_l}{F_{m1}^* + B_{m1}^* - r_l} = \frac{\overline{U} - r_l}{p^o}, B_{m1}^* = r_l + \frac{(I - A)(\overline{U} - r_l)}{p^o (p^o + r_l - \overline{U})}, \]

\[ F_{m1}^* = \frac{I - A}{p^o}, \quad p_{m1}^* = p^o. \]

In equilibrium the first-best risk level is achieved. Also, the creditor breaks even \((U_C = 0)\) and the manager’s expected payoff is exactly his reservation utility \((U_M = \overline{U})\). Notice that the equilibrium
inside debt $B_{m1}^*$ in (6) is indeed larger than $r_l$, which confirms our initial conjecture.

**The case of $U \leq r_l$:** In the other case in which the manager’s reservation utility is smaller than the liquidation value ($U \leq r_l$), because the manager has the seniority, the manager’s reservation utility will be satisfied even when the project fails and is liquidated. We conjecture that the inside-debt compensation in equilibrium is smaller than the liquidation value; that is, $\min\{r_l, B\} = B$ or $r_l > B \geq U$. Solving for the equilibrium, we have

$$
\alpha_{m2}^* = \frac{B_{m2}^* - B_{m2}}{F_{m2}^* + B_{m2}^* - r_l} = 0, \quad B_{m2}^* = \bar{U},
$$

(7)

$$
F_{m2}^* = \frac{(I - A) - (1 - p^*) \cdot (r_l - \bar{U})}{p^*}, \quad p_{m2}^* = p^*.
$$

Again, the first-best risk level $p^*$ is induced, the creditor breaks even ($U_C = 0$) and the manager’s expected payoff is exactly his reservation utility in equilibrium. Notice that the equilibrium inside debt $B_{m2}^*$ in (7) is indeed lower than $r_l$, which confirms our initial conjecture.

We summarize the equilibrium results of these two cases in the manager-seniority scenario in the following proposition.

**Proposition 3** In the manager-seniority scenario,

- if $U > r_l$, in equilibrium the firm offers $\alpha_{m1}^* = \frac{B_{m1}^* - \min\{r_l, B_{m1}^*\}}{F_{m1}^* + B_{m1}^* - r_l} = \frac{\bar{U} - r_l}{p^*}$ and $B_{m1}^* = r_l + \frac{(I - A)(\bar{U} - r_l)}{p^* \cdot (p^* + r_l - \bar{U})}$, the creditor requests $F_{m1}^* = \frac{I - A}{p^*}$, and the manager chooses $p_{m1}^* = p^*$;

- if $U \leq r_l$, in equilibrium the firm offers $\alpha_{m2}^* = \frac{B_{m2}^* - \min\{r_l, B_{m2}^*\}}{F_{m2}^* + B_{m2}^* - r_l} = 0$ and $B_{m2}^* = \bar{U}$, the creditor requests $F_{m2}^* = \frac{(I - A) - (1 - p^*) \cdot (r_l - \bar{U})}{p^*}$, and the manager chooses $p_{m2}^* = p^*$.

Again, the firm offers both performance-based and inside-debt compensation to calibrate the manager’s risk choice to be the first-best risk level, and in equilibrium the firm achieves the first-best expected payoff $\Pi^0$. In this scenario, the “right mix” of the compensation package should satisfy $\alpha = \frac{B - \min\{r_l, B\}}{F + B - r_l}$ to induce the first-best risk choice. Notice that in the manager-seniority scenario, the proportion of equity as compensation is lower than the inside-debt proportion in the equal-seniority scenario ($\alpha = \frac{B - \min\{r_l, B\}}{F + B - r_l} < \frac{B}{F + B}$). This is because when the manager has the seniority, the manager will receive $\min\{r_l, B\}$ upon bankruptcy. As he is well protected upon bankruptcy he
tends to take more risk. To induce the manager to take the lower first-best risk level, the firm must offer a lower portion of equity as the performance-based compensation. In addition, in equilibrium the firm choose $\alpha$ and $\beta$ that not only satisfy the “right mix” but also make the manager’s expected payoff exactly his reservation utility. Proposition 3 shows that in both cases of $\bar{U} > r_l$ and $\bar{U} \leq r_l$, the equilibrium inside-debt compensation are positive.

**Corollary 3** In the manager-seniority scenario,

- the firm offers a compensation package that satisfies $\alpha = \frac{B - \min\{r_l, B\}}{F + B - r_l}$ to induce the manager to choose $p^0$;

- in equilibrium the inside-debt compensation in the compensation package is positive ($B^*_{m1} > 0$ and $B^*_{m2} > 0$).

It is interesting to note that regardless of the seniority order of the debts in these scenarios, the firm can always achieve the first best by choosing the “right mix” in the managerial compensation package. This implies that even if the seniority order of the debts is an endogenous choice of the firm (or its shareholders), our analysis will remain the same because the firm is indifferent among the three seniority orders.

5 Comparisons among Scenarios

In this section, we compare the equilibrium choices across the three scenarios to analyze how debt seniority influences the firm’s and the creditor’s strategies. We also compare some comparative statics among the three scenarios, which have interesting implications.

5.1 Comparison of the Equilibrium Inside Debts, Performance-based Compensations, and Requested Repayments in the Three Scenarios

We first compare the equilibrium choices across the three scenarios to analyze how debt seniority influences the firm’s and the creditor’s strategies. For convenience, we list the equilibrium $\alpha^*, B^*$,
and $F^*$ in the three scenarios in the following table.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Equal-seniority Scenario</th>
<th>Creditor-seniority Scenario</th>
<th>Manager-seniority Scenario</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha^*$</td>
<td>$\frac{p}{p^0+r_l}$</td>
<td>$\frac{p}{p^0}$</td>
<td>if $\bar{U} &gt; r_l$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>if $\bar{U} \leq r_l$</td>
</tr>
<tr>
<td>$B^*$</td>
<td>$\frac{U(I-A)}{p^0(p^0+\bar{U})} - \frac{U_l(l-\rho^0)}{p^0(p^0+\bar{U})}$</td>
<td>$\frac{U(I-A-r_l)}{p^0(\bar{U}-r_l)}$</td>
<td>$r_l + \frac{(I-A)(\bar{U}-r_l)}{p^0(\bar{U}-r_l)}$</td>
</tr>
<tr>
<td>$F^*$</td>
<td>$\frac{I-A-(1-\rho^0)r_l}{p^0} + \frac{U_l(l-\rho^0)}{p^0(\bar{U}+r_l)}$</td>
<td>$\frac{I-A-(1-\rho^0)r_l}{p^0}$</td>
<td>$\frac{I-A}{p^0}$</td>
</tr>
</tbody>
</table>

The fundamental difference among these scenarios is how the liquidation value $r_l$ is split between the manager and the creditor upon bankruptcy. When the liquidation value approaches zero, it is intuitive that the equilibrium strategies ($\alpha^*$, $B^*$, and $F^*$) are identical across all the three scenarios (note that the manager-seniority scenario with $\bar{U} \leq r_l$ is not applicable). That is, with $r_l \to 0$, regardless of the debt seniority, the equilibrium strategies are

$$\alpha^* = \frac{\bar{U}}{p^0}; \quad B^* = \frac{(I-A)\bar{U}}{p^0(p^0 - \bar{U})}; \quad F^* = \frac{I-A}{p^0}.$$

When the liquidation value is positive, the equilibrium varies with the debt seniority.

We start with the comparison of the equilibrium inside debts:

**Proposition 4** When $\bar{U} > r_l$,

- **(Case I-1):** $B^*_{M1} > B^*_c > B^*_c$ if $I - A < \frac{[p^0+\bar{U}(1-\rho^0)+\rho^0 r_l](\rho^0-\bar{U}+r_l)}{p^0+r_l}$;
- **(Case I-2):** $B^*_c > B^*_c > B^*_{M1}$ if $\frac{[p^0+\bar{U}(1-\rho^0)+\rho^0 r_l](\rho^0-\bar{U}+r_l)}{p^0+r_l} < I - A < \frac{[p^0+\bar{U}(1-\rho^0)](\rho^0-\bar{U}+r_l)}{p^0}$;
- **(Case I-3):** $B^*_c > B^*_c > B^*_{M1}$ if $\frac{[p^0+\bar{U}(1-\rho^0)](\rho^0-\bar{U}+r_l)}{p^0+r_l} < I - A < \frac{[p^0+\bar{U}(1-\rho^0)+r_l](\rho^0-\bar{U}+r_l)}{p^0+r_l}$;
- **(Case I-4):** $B^*_c > B^*_c > B^*_{M1}$ if $I - A > \frac{[p^0+\bar{U}(1-\rho^0)+r_l](\rho^0-\bar{U}+r_l)}{p^0+r_l}$.

When $\bar{U} \leq r_l$,

- **(Case II-1):** $B^*_{M2} > B^*_c > B^*_c$ if $I - A < \frac{(p^0+r_l)(\rho^0-\bar{U}+r_l)}{p^0+r_l}$;
- **(Case II-2):** $B^*_c > B^*_c > B^*_{M2}$ if $\frac{(p^0+r_l)(\rho^0-\bar{U}+r_l)}{p^0+r_l} < I - A < p^0(\rho^0 - \bar{U}) + r_l$;
- **(Case II-3):** $B^*_c > B^*_c > B^*_{M2}$ if $p^0(\rho^0 - \bar{U}) + r_l < I - A < \frac{[p^0+\bar{U}(1-\rho^0)+r_l](\rho^0-\bar{U}+r_l)}{p^0+r_l}$.
• (Case II-4): $B_e^* > B_e^* > B_{M2}^*$ if $I - A > \frac{(p^o+\bar{U}(1-p^o)+r_l)(p^o-\bar{U}+r_l)}{p^o+r_l}$.

Proposition 4 shows that, when the borrowing amount $I - A$ is small, the inside-debt compensation in the manager-seniority scenario ($B_{M1}^*$ or $B_{M2}^*$) is the highest and the inside-debt compensation in the creditor-seniority scenario ($B_c^*$) is the lowest. Intuitively, when the creditor has the seniority, the manager tends to take less risk because he receives no payment upon failure. To motivate the manager to take the higher first-best risk level, the firm must offer a higher performance-based compensation $\alpha_c^*$ and a lower inside-debt compensation $B_c^*$. On the other hand, when the manager has the seniority, the manager tends to take more risk because he is well protected upon failure. To motivate the manager to take the lower first-best risk level, the firm offers a lower performance-based compensation $\alpha_{M1}^*$ and a higher inside-debt compensation $B_{M1}^*$ or $B_{M2}^*$.

As $I - A$ increases, our analysis shows that the inside-debt compensation in all three scenarios increase, but the inside-debt compensation in the manager-seniority scenario, $B_{M1}^*$ or $B_{M2}^*$, increases at the slowest pace. In other words, $B_{M1}^*$ or $B_{M2}^*$ is influenced the least by the larger loan size. As a result, when $I - A$ is sufficiently large, the inside-debt compensation in the manager-seniority scenario becomes the smallest among those in the three scenarios. This is because when the loan size is large, most of the project return will be paid to the creditor as the repayment, and the profit left for the firm will be small; hence, the manager’s performance-based compensation is very limited as the performance-based compensation is proportional to the firm’s profit after debt liabilities. Therefore, the firm must increase the inside-debt compensation to satisfy the manager’s reservation utility in all the three scenarios. However, in the manager-seniority scenario the firm does not need to increase the inside-debt compensation so much as in the other two scenarios, because the manager is better protected when he has the seniority.

The results in Proposition 4 also show that we have $B_c^* < B_e^*$ when $I - A$ is small, but when $I - A$ becomes sufficiently large, we have $B_c^* > B_e^*$. As we discussed before, the firm uses both performance-based compensation and inside-debt compensation to induce the choice of the first-best risk level. In the creditor-seniority scenario, the manager obtains no compensation from inside debt upon project failure and only gets compensated when the project is successful. Because the inside-debt compensation is unable to protect the manager from the bankruptcy risk, the manager tends to choose lower risk. In order to induce the manager to choose the higher first-best risk
level, the firm offers a higher performance-based compensation and in general a lower inside-debt compensation than that in the equal-seniority scenario; i.e., $\alpha^*_c > \alpha^*_e$ and $B^*_c < B^*_e$.

However, it is not always the case that $B^*_c < B^*_e$. When the loan size $I - A$ is very large, again, the creditor requests for a relatively large repayment compared with the project cash flow, and the equity value after repayment is very low even upon project success. To satisfy the manager’s reservation utility, the firm must provide high inside debt compensation to the manager. In the equal-seniority scenario, the manager may still get some of the liquidity value upon failure, while in the creditor-seniority scenario the manager gets no inside-debt payment upon failure. Therefore, the firm needs to offer an even higher inside-debt compensation in the creditor-seniority scenario to satisfy the manager’s reservation utility.

We also compare $\alpha^*$ and $F^*$ across the three scenarios, and we have the following results.

**Proposition 5**

- $\alpha^*_c > \alpha^*_e > \alpha^*_{m1} > \alpha^*_{m2} = 0$;
- $F^*_c < F^*_e < F^*_{m2} < F^*_{m1}$.

As we already discussed, $\alpha^*_c > \alpha^*_e$ is because in the creditor-seniority scenario the manager tends to choose lower risk and the firm must offer a higher performance-based compensation to induce the higher first-best risk level. Proposition 5 also indicates that the performance-based compensation in the equal-seniority scenario is higher than that in the manager-seniority scenario ($\alpha^*_e$ is larger than $\alpha^*_{m1}$, $\alpha^*_{m2}$). This is because in the manager-seniority scenario the manager receives inside-debt compensation before any repayment to the creditor upon project failure. The manager is well protected by the inside-debt compensation from the bankruptcy risk and tends to choose higher risk. In other words, a marginal increase in the risk level (i.e., a marginal decrease in $p$) is more beneficial to the manager in the manager-seniority scenario than in the equal-seniority scenario. To induce the manager to choose the lower first-best risk level, the firm must offer a lower performance-based compensation than that in the equal-seniority scenario. In addition, we always have $\alpha^*_{m1} > \alpha^*_{m2}$; this is because when the liquidation value is high ($U \leq r_l$), the manager receives a fixed payment from inside-debt compensation and no performance-based compensation ($\alpha^*_{m2} = 0$).

Regarding the comparison of $F^*$ across scenarios, we find that in the creditor-seniority scenario, the creditor requests for a lower repayment than in the equal-seniority scenario (i.e., $F^*_c < F^*_e$). This
is because upon project failure, all the liquidation value \( r_l \) will be paid to the creditor, and thus the creditor is better protected from bankruptcy risk than in the equal-seniority scenario. In addition, our analysis shows that the creditor asks for a lower repayment in the equal-seniority scenario than in the manager-seniority scenario. The reason is that in the manager-seniority scenario, the liquidation value does not help to protect the creditor from bankruptcy risk due to the manager’s inside-debt seniority. The creditor may only receive the repayment upon project success and is exposed to the risk of bankruptcy. As a result, the creditor must request a higher debt repayment to cover the bankruptcy risk compared with in the equal-seniority scenario. Furthermore, we have \( F_{m2}^* < F_{m1}^* \) because when the liquidation value is high (\( \bar{U} \leq r_l \)), upon bankruptcy, there is still some remaining balance paid to the creditor after the firm paying off the inside debt. In other words, the creditor is protected by the high liquidation value and thus requests a lower debt repayment compared with the case when the liquidation value is low (i.e., when \( \bar{U} > r_l \)).

### 5.2 Comparison of Comparative Statics

We now analyze some comparative statics which provide interesting implications. Specifically, we examine how the liquidation value \( r_l \) and the manager’s reservation utility \( \bar{U} \) affect the compensation package and the requested repayment in equilibrium.

We first examine how the liquidation value affects equilibrium repayment and compensation. Our analysis shows the following properties.

**Corollary 4**  
- **In the equal-seniority scenario,**
  
  - If \( I - A \) is sufficiently large, \( F_e^* \) increases in the liquidation value \( r_l \) \( (\frac{dF_e^*}{dr_l} > 0) \); otherwise, \( F_e^* \) decreases in \( r_l \) \( (\frac{dF_e^*}{dr_l} < 0) \);
  
  - \( \alpha_e^* \) decreases in \( r_l \); \( B_e^* \) may increase or decrease in \( r_l \).

- **In the creditor-seniority scenario,**
  
  - If \( I - A \) is sufficiently large, \( F_c^* \) increases in the liquidation value \( r_l \) \( (\frac{dF_c^*}{dr_l} > 0) \); otherwise, \( F_c^* \) decreases in \( r_l \) \( (\frac{dF_c^*}{dr_l} < 0) \), \( \frac{dF_c^*}{dr_l} < \frac{dF_e^*}{dr_l} \);
  
  - \( \alpha_c^* \) increases in \( r_l \); \( B_c^* \) may increase or decrease in \( r_l \).
In the manager-seniority scenario,

- \( F^*_{m1} \) and \( F^*_{m2} \) increase in \( r_l \) (\( \frac{dF^*_{m1}}{dr_l} > 0 \) and \( \frac{dF^*_{m2}}{dr_l} > 0 \));
- if \( U > r_l \), \( \alpha^*_m \) decreases in \( r_l \); \( B^*_m \) may increase or decrease in \( r_l \); if \( U \leq r_l \), \( \alpha^*_m \) and \( B^*_m \) do not change with \( r_l \).

Conventional wisdom may believe that when the liquidation value \( r_l \) is higher, the creditor’s downside risk would be better protected and thus the requested repayment would always decrease. However, Corollary 4 shows that the requested repayment \( F^* \) may increase or decrease with the liquidation value; in addition, in the manager-seniority scenario, the requested repayment in fact always increases in the liquidation value, which is just the opposite of conventional wisdom. Our analysis reveals that, once we consider inside-debt compensation, there is another side of the coin. The liquidation value actually gives rise to two countervailing effects on the creditor’s decision of repayment \( F^* \):

- **Protection-upon-failure effect.** A higher liquidation value \( r_l \) protects the creditor’s downside risk upon project failure, thereby resulting in a lower debt repayment \( F \).

- **Risk-taking effect.** As the liquidation value \( r_l \) increases, the incremental loss from taking more risk is lower and thus the equilibrium risk level becomes higher. To break even the creditor needs to request a higher repayment \( F \).

In the equal-seniority scenario, which of the two countervailing effects dominates depends on the amount of borrowing, or the loan size \((I - A)\). If the firm borrows a lot from the creditor \((I - A \text{ is large})\), as the liquidation value \( r_l \) increases, the creditor’s downside risk is protected a little bit better, but the creditor worries much more about the manager taking more risk, because a higher-risk choice implies the creditor is likely to suffer the loss of a large amount of lending. In other words, the risk-taking effect dominates the protection-upon-failure effect in the creditor’s consideration. As a result, the creditor requests a higher repayment \( F^*_c \) (i.e., \( F^*_c \) increases in \( r_l \)). On the other hand, if the borrowing amount is small, the protection-upon-failure effect dominates, and the creditor requests a lower repayment as the liquidation value increases.

In the creditor-seniority scenario, again, whether the requested repayment \( F^*_c \) increases or decreases with the liquidation value depends on the borrowing amount. However, in the creditor-
seniority scenario, it is more likely that the requested repayment decreases in the liquidation value than in the equal-seniority scenario \( \left( \frac{dF^*_e}{dr_l} > \frac{dF^*_c}{dr_l} \right) \). This is because when the creditor has the seniority to claim the liquidation value upon bankruptcy, the creditor is protected better and thus the protection-upon-failure effect is relatively stronger than in the equal-seniority case. As a result, it is more likely that the protection-upon-failure effect dominates and therefore \( F^*_c \) decreases in \( r_l \).

In the manager-seniority scenario, the requested repayment \( (F^*_m_1 \text{ or } F^*_m_2) \) always increases in the liquidation value \( r_l \). This is because upon bankruptcy the manager’s inside debt is settled before any repayment to the creditor, and thus the protection-upon-failure effect of higher \( r_l \) for the creditor is negligible. On the other hand, the risk-taking effect still exists; when the liquidation value \( r_l \) gets higher, the equilibrium risk level becomes higher (lower \( p^0 \)). Anticipating that the manager will choose higher risk, the creditor must request a higher repayment to break even \( (F^*_m_1 \text{ and } F^*_m_2 \text{ increase in } r_l) \).

In all three scenarios, as the requested repayment \( F^* \) changes with the liquidation value \( r_l \), \( \alpha^* \) and \( B^* \) change accordingly to satisfy two conditions: first, \( \alpha^*, B^* \) and \( F^* \) must satisfy the “right mix” so that the first-best risk level is induced; second, \( \alpha^* \) and \( B^* \) must make the manager’s expected payoff to be exactly his reservation utility \( U \).

We also examine the comparative statics regarding the manager’s reservation utility, and we have the following results:

**Corollary 5**

- In the equal-seniority scenario, \( F^*_e, \alpha^*_e, \) and \( B^*_e \) increase in the manager’s reservation utility \( U \).

- In the creditor-seniority scenario, \( \alpha^*_c \) and \( B^*_c \) increase in \( U \), and \( F^*_c \) does not change with \( U \).

- In the manager-seniority scenario, if \( U > r_l \), \( \alpha^*_m_1 \) and \( B^*_m_1 \) increase in \( U \), \( F^*_m_1 \) does not change with \( U \); if \( U \leq r_l \), \( \alpha^*_m_2 \) does not change with \( U \), \( F^*_m_2 \) and \( B^*_m_2 \) increase in \( U \).

In the equal-seniority scenario, we find that \( F^*_e, \alpha^*_e, \) and \( B^*_e \) increase in the manager’s reservation utility \( U \). To maximize its expected payoff, the firm chooses the compensation package so that the manager’s expected compensation equals \( U \). If the manager’s reservation utility gets higher, the firm must increase both the performance-based compensation and inside-debt compensation to satisfy the higher reservation utility (i.e., \( \alpha^*_e \) and \( B^*_e \) increase in \( U \)). Since the inside-debt compensation
$B^*_e$ increases in $\bar{U}$, the manager obtains a larger payoff from inside debt upon project failure when $\bar{U}$ is higher. As a result, the creditor obtains a lower payoff upon project failure. To break even, the creditor requests a higher repayment to cover the risk of project failure; i.e., $F^*_e$ increases in $\bar{U}$.

In the creditor-seniority scenario the debt repayment $F^*_e$ does not change with the reservation utility $\bar{U}$. This is because when the creditor has the seniority, upon project failure all the liquidation value $r_l$ is paid to the creditor, and the creditor’s payoff is not affected by the manager’s compensation plan. No matter how $\alpha^*_c$ and $B^*_e$ change with $\bar{U}$, the creditor’s expected payoff is not affected and thus there is no impact on the creditor’s requested repayment $F^*_e$.

In the manager-seniority scenario, if $\bar{U} > r_l$, the requested repayment $F^*_{m1}$ does not change with the manager’s reservation utility $\bar{U}$, because all the liquidation value $r_l$ is paid to the manager upon project failure, and the creditor’s payoff is not affected by the manager’s compensation plan. In the case of $\bar{U} \leq r_l$, the manager’s compensation is a fixed payment with $\alpha^*_{m2} = 0$ and $B^*_{m2} = \bar{U}$. It is obvious that $\alpha^*_{m2}$ does not change with $\bar{U}$ and $B^*_{m2}$ increases in $\bar{U}$. When $\bar{U}$ gets higher, the manager obtains more payoff from inside debt and the creditor obtains less payoff upon project failure. To break even, the creditor requests a higher repayment; i.e., $F^*_{m2}$ increases in $\bar{U}$.

6 Discussions on Risk Averse Manager and Empirical Implications

6.1 Risk Averse Manager

In our main setting, we show that the firm can achieve the optimal risk level in equilibrium and reach the maximum expected payoff in all scenarios. This is because the firm can induce the manager to pick the optimal risk level $p^o$ by choosing the “right mix” and the expected compensation to the manager is exactly equal to the reservation utility due of the risk neutrality. Our focus is to analyze how the seniority orders may simultaneously affect the optimal levels of equity-based compensation and inside debt compensation in equilibrium.

Alternatively, one may consider the case in which the manager is risk averse. With a risk averse manager, the firm has to consider both the induced risk level and the associated risk premium compensated to the manager. On one hand, the firm intends to induce the risk-averse manager to

---

$^6$ When $\bar{U} > r_l$, the inside-debt compensation is larger than the liquidation value ($B^*_{m1} > r_l$) as shown in Proposition 3.
choose the optimal risk level $p^o$ to maximize the expected project cash flow as in the risk neutral setup. On the other hand, the firm has to compensate the manager for his risk premium, which is related to the project risk level. Thus the firm faces an additional economic trade-off between the benefit of inducing a higher risk level and the cost of paying higher risk premium to the manager.

We can show that in equilibrium, the firm is better off to induce the manager to pick a risk level lower than the optimal level to save risk premium. The firm’s ex ante payoff in the risk averse setting is lower than that in the risk neutral setting for two reasons: (i) the project risk level is lower than the optimal level, which leads to lower the expected project cash flow; (ii) the firm has to compensate the manager for his risk premium.

The risk premium varies across the three scenarios of different seniority orders. Among the three scenarios, the manager’s risk premium is the highest in the creditor-seniority scenario, because upon bankruptcy the manager receives nothing and the creditor obtains the full amount of the liquidation value. As a result, to save risk premium, the equilibrium risk level is the lowest in the creditor-seniority scenario. Due to the lowest risk level (i.e., the equilibrium risk level is way lower than the optimal level, which leads to the lowest project cash flow) and the highest risk premium, the firm’s ex ante payoff is the lowest in the creditor-seniority scenario. In contrast, the manager’s risk premium is the lowest in the manager-seniority scenario, because upon bankruptcy the manager is well protected and his inside debt is settled before any repayment to the creditor. As a result, the equilibrium risk level is the highest and the firm’s ex ante payoff is the highest in the manager-seniority scenario. Intuitively, the equal-seniority scenario is right in between the other two scenarios.

It is worth emphasizing that with a risk-averse manager, the firm can still achieve the first-best result $p^o$ and incur no risk premium in the manager-seniority scenario when the manager’s reservation utility is smaller than the liquidation value ($U_l < r_l$). The reason is that if $U_l \leq r_l$, upon bankruptcy the liquidation value is high enough to cover the manager’s reservation utility, the firm offers zero equity compensation ($\alpha = 0$) and inside debt as the amount of reservation utility ($B = U_l$) (the same as in the risk neutral setting). Since the inside debt can always be fully paid regardless of the project outcome, the risk-averse manager bears no risk from his compensation and no risk premium is incurred. Without any risk premium, the firm is better off by inducing the optimal risk level $p^o$ and the first best result is achieved in this scenario. This result again
highlights the importance of seniority orders to inside debt compensation.

6.2 Empirical Implications

Our model provides some predictions regarding inside-debt compensation that future empirical studies may test. For example, our analysis shows that, when the borrowing amount is small, the inside-debt compensation in the manager-seniority scenario is the highest and the inside-debt compensation in the creditor-seniority scenario is the lowest. However, when the borrowing amount is sufficiently large, the opposite is true; that is, the inside-debt compensation in the manager-seniority scenario is lower. Future research can test our predictions by two ways. First, one may look at creditor-seniority firms and test whether firms with lower leverage offer less inside-debt compensation such as pension or deferred benefits to their managers. Second, one may examine firms with similar leverage and compare firms with different debt seniority orders. According to our analysis, for a sample of firms with low leverage, firms with creditor seniority will offer less pension than firms with manager seniority, while for a sample of very-high-leverage firms we will find the opposite result.

Our analysis also shows that when external debts have seniority, managers tend to choose lower risk, and firms must provide higher performance-based compensation to induce the managers to choose the higher first-best risk level. In contrast, when managers’ inside debts have seniority, managers tend to take more risk because they are well protected upon failure. To induce the lower first-best risk level, firms offer lower performance-based compensation. These can be tested empirically by examining the relationship between debt seniority and performance-based compensation.

We also analytically compare the debt repayments across scenarios. It is intuitive that the creditor asks for a lower repayment when they have seniority than managers. This is because upon project failure, all the liquidation value will be paid to the creditor, and thus the creditor is better protected for bankruptcy risk than in the creditor-seniority scenario. Anantharaman et al. (2014) analyze the effects of the seniority of inside debt compensation on debt contracting. They use the duration of inside debt compensation as a proxy for seniority, and they find that controlling for relative leverage, promised loan spreads are smaller when inside debt is less senior to external debt. This empirical result supports our prediction. However, they do not directly test the effects of seniority and leverage on the mix of inside-debt and performance-based compensation, which may
be interesting for future studies to examine.

In addition, our study analyzes the relationship between the liquidation value and the requested repayment. We show that in both the equal-seniority and creditor-seniority scenarios, the requested repayment increases in the liquidation value when the borrowing amount is large, and decreases in the liquidation value otherwise. However, the requested repayment is more likely to decrease in the liquidation value in the creditor-seniority case. In the manager-seniority scenario, the requested repayment always increases in the liquidation value. These analytical predictions can be tested empirically. For example, one may study companies with equal- or creditor-seniority, and test whether in a subsample of high-leverage companies the association between debt financing costs and liquidation values is positive, while in a subsample of low-leverage companies the association is negative. One may also test whether this association is always positive in a sample of companies with manager seniority.

7 Concluding Remarks

In this paper we study firms’ endogenous reasons of offering inside-debt compensation. Some previous studies on inside debts regard inside-debt compensation as measures to align the interests between firms and debtholders, and it is usually argued that regulators should impose regulations enforcing firms to offer inside-debt compensation to protect debtholders’ welfare. In contrast, our study shows that firms actually do have an incentive to offer inside-debt compensation besides performance-based compensation. This is because by choosing the “right mix” between the performance-based compensation and the inside-debt compensation, a firm is able to calibrate its manager’s risk choice to maximize the firm’s (or the shareholders’) welfare and achieve the first best. Without inside-debt compensation, the firm is unable to achieve the first-best optimality.

Our study sheds light on the popularity of voluntary inside-debt compensation, and provides insights for regulators when considering regulating pensions and deferred compensation. Our analysis also provides empirical implications for future empirical studies on inside debts.
References


Appendix I: An Analysis with a General Function Form of $R(p)$

In this appendix we show that our main results still hold with a general function form of $R(p)$, which decreases in $p$. We first analyze the first-best benchmark and then the three scenarios.

(1) First-best Benchmark

With a general $R(p)$, the firm’s ex-ante expected payoff is

$$\Pi^o \equiv p \cdot R(p) + (1 - p)r_l - I - U,$$

and the first-order condition to solve for $p^o \in (0, 1)$ is

$$\frac{\partial \Pi^o}{\partial p} = R(p) - r_l + p \frac{\partial R(p)}{\partial p} = 0. \quad (8)$$

(2) Equal-seniority Scenario

In the equal-seniority scenario,

$$U_M = \alpha \cdot p \cdot [R(p) - F - B] + p \cdot B + (1 - p)r_l \frac{B}{F + B}.$$

The first-order condition shows the induced risk level $p^e$ solves

$$\frac{\partial U_M}{\partial p} = \alpha [R(p) - F - B] + B - r_l \frac{B}{F + B} + \alpha \cdot p \frac{\partial R(p)}{\partial p} = 0, \quad (9)$$

which can be rewritten to be

$$\alpha \left[ R(p) - r_l + p \frac{\partial R(p)}{\partial p} \right] + \left( \frac{B}{F + B} - \alpha \right) \frac{B}{F + B} = 0, \quad (9)$$

From the above equation, one can easily see that when $\alpha = \frac{B}{F + B}$ the first-best first-order condition is congruent with the manager’s first-order condition, which ensures that the first-best risk level is induced. Therefore, the “right mix” to induce the first-best risk choice is

$$\alpha = \frac{B}{F + B}. \quad (9)$$
The manager’s expected payoff should equal his reservation utility,

\[ U_M = \alpha \cdot p^0 \cdot [R(p^o) - F - B] + p^0 \cdot B + (1 - p^o)r_l \frac{B}{F + B} = \bar{U}. \] (10)

Substituting \( \alpha = \frac{B}{F + B} \) into the above IR constraint, we have

\[ \frac{B}{F + B} \cdot p^0 \cdot [R(p^o) - F - B] + p^0 \cdot B + (1 - p^o)r_l \frac{B}{F + B} = \bar{U}, \]
\[ \frac{B}{F + B} \cdot [p^0 \cdot R(p^o) + (1 - p^o)r_l] = \bar{U}. \]

Therefore, we have

\[ \alpha^* = \frac{\bar{U}}{p^0 \cdot R(p^o) + (1 - p^o)r_l} > 0, \]
\[ B^*_e = \frac{\alpha^*}{1 - \alpha^*} F^* > 0. \]

(3) Creditor-seniority Scenario

In the creditor-seniority scenario,

\[ U_M = \alpha \cdot p \cdot [R(p) - F - B] + p \cdot B. \]

The manager’s first-order condition of his risk choice is

\[ \frac{\partial U_M}{\partial p} = \alpha [R(p) - F - B] + B + \alpha \cdot p \frac{\partial R(p)}{\partial p} = 0, \]

which can be written into

\[ \alpha \left[ R(p) - r_l + p \frac{\partial R(p)}{\partial p} \right] + \left( \frac{B}{F + B - r_l} - \bar{U} \right) = 0. \]

From the above equation, one can easily see that when \( \alpha = \frac{B}{F + B - r_l} \) the first best risk level is
induced. That is, the “right mix” in this scenario is

$$\alpha = \frac{B}{F + B - r_l}.$$ 

Substituting $\alpha = \frac{B}{F + B - r_l}$ into the manager’s IR constraint,

$$U_M = \alpha \cdot p^o \cdot [R(p^o) - F - B] + p^o \cdot B = \overline{U},$$

we have

$$\frac{B}{F + B - r_l} \cdot p^o \cdot [R(p^o) - F - B] + p^o \cdot B = \overline{U},$$

$$\frac{B}{F + B - r_l} \cdot p^o \cdot R(p^o) - \frac{B \cdot p^o \cdot r_l}{F + B - r_l} = \overline{U},$$

$$\frac{B}{F + B - r_l} \cdot p^o \cdot [R(p^o) - r_l] = \overline{U}.$$

Therefore,

$$\alpha^*_c = \frac{\overline{U}}{p^o \cdot [R(p^o) - r_l]} > 0,$$

$$B^*_c = \frac{\alpha^*_c (F^*_c - r_l)}{1 - \alpha^*_c} > 0.$$

(4) Manager-seniority Scenario

In the manager-seniority scenario,

$$U_M = \alpha \cdot p \cdot (R(p) - F - B) + p \cdot B + (1 - p) \min\{r_l, B\},$$

and the manager’s first-order condition of his risk decision is

$$\frac{\partial U_M}{\partial p} = \alpha [R(p) - F - B] + B - \min\{r_l, B\} + \alpha \cdot p \frac{\partial R(p)}{\partial p} = 0.$$
which can be rearranged to be
\[
\alpha \left[ R(p) - r_l + p \frac{\partial R(p)}{\partial p} \right] + \left( \frac{B - \min\{r_l, B\}}{F + B - r_l} - \alpha \right) (F + B - r_l) = 0.
\]

From the above equation, we can see that the first best risk level is induced when
\[
\alpha = \frac{B - \min\{r_l, B\}}{F + B - r_l}.
\]

It is obvious that in equilibrium, we must have \( \alpha \geq 0 \) in equilibrium.

Substituting \( \alpha = \frac{B - \min\{r_l, B\}}{F + B - r_l} \) into the manager’s IR constraint
\[
U_M = \alpha \cdot p^\circ \cdot [R(p^\circ) - F - B] + p^\circ \cdot B + (1 - p^\circ) \min\{r_l, B\} = \mathcal{U},
\]

we have
\[
\frac{B - \min\{r_l, B\}}{F + B - r_l} \cdot p^\circ \cdot [R(p^\circ) - F - B + r_l] + p^\circ \cdot B + (1 - p^\circ) \min\{r_l, B\} - r_l \frac{B - \min\{r_l, B\}}{F + B - r_l} = \mathcal{U},
\]
\[
\frac{B - \min\{r_l, B\}}{F + B - r_l} \cdot p^\circ \cdot R(p^\circ) + \min\{r_l, B\} - (1 - p^\circ) \cdot (B - \min\{r_l, B\}) - r_l \frac{B - \min\{r_l, B\}}{F + B - r_l} = \mathcal{U},
\]
\[
(B - \min\{r_l, B\}) \left[ \frac{p^\circ \cdot R(p^\circ) - r_l}{F + B - r_l} - (1 - p^\circ) \right] + \min\{r_l, B\} = \mathcal{U} > 0.
\]

Thus we must have \( B_m^* > 0 \).

**Appendix II: The Case of Enforcing \( B = 0 \)**

If we enforce \( B = 0 \), the manager’s first-order condition for his choice of risk is \( \frac{\partial U_M}{\partial p} = \alpha [R(p) - F - 1] = 0 \), which gives \( p_{B=0} = \exp[r_f - 1 - F] \). From the creditor’s break-even condition, the requested repayment is \( F_{B=0} = \frac{(1-A) \cdot (1-p) r_l}{p} \). Combining these two equations together, we have \( \ln p_{B=0} + \frac{I-A-r_l}{p_{B=0}} = r_f - 1 - r_l \). Because \( \frac{I-A-r_l}{p_{B=0}} > 0 \), it follows that \( \ln p_{B=0} = r_f - 1 - r_l - \frac{I-A-r_l}{p_{B=0}} < r_f - 1 - r_l = \ln p^\circ \).

That is, the manager chooses a risk level higher than the first-best risk level \( (p_{B=0} < p^\circ) \). Note that \( p_{B=0} \) does not depend on \( \alpha \); that is, the manager always chooses \( p_{B=0} \) regardless of what
performance-based compensation the firm offers. In equilibrium, the firm chooses \( \alpha \) to satisfy the manager’s IR constraint: \( \alpha \cdot p_{B=0} \cdot (R(p_{B=0}) - F_{B=0}) = \overline{U} \), which gives \( \alpha_{B=0} = \frac{\overline{U}}{p_{B=0}} \). With \( \alpha = \frac{\overline{U}}{p_{B=0}} \), the firm’s ex-ante payoff is

\[
(1 - \alpha) \cdot p_{B=0} \cdot (R(p_{B=0}) - F_{B=0}) - A,
\]

\[
= p_{B=0} - \overline{U} - A,
\]

\[
< p^o + r_l - I - \overline{U} = \Pi^o.
\]

That is, the firm’s ex-ante expected payoff in this enforcing \( B = 0 \) case is lower than the firm’s expected payoff in the first best.

**Appendix III: Proofs**

**Proof of Proposition 1, Corollary 1, and part of Corollaries 4 and 5**

From the analysis in the context, (5) shows that a compensation package that satisfies \( \alpha = B/(F + B) \) motivates the manager to choose \( p^o \). When \( p^o \) is achieved, the firm expected payoff equals to \( \Pi_F \). Given the optimal risk level \( p^o \) and \( \alpha = B/(F + B) \), the firm’s problem is to select \( F \) and \( B \) in order to satisfy

\[
U_C = p^o \cdot F + (1 - p^o) \cdot r_l \frac{F}{F + B} - (I - A) = 0,
\]

\[
U_M = \alpha \cdot p^o \cdot (R(p^o) - F - B) + p^o \cdot B + (1 - p^o) r_l \frac{B}{F + B} = \overline{U},
\]

where \( R(p^o) = r_l + 1 \), or \( p^o = \exp[r_f - r_l - 1] \).

The first-order condition for the manager to choose the risk level is

\[
\frac{\partial U_M}{\partial p} = \alpha [R(p) - F - B - 1] + B - r_l \frac{B}{F + B} = 0.
\]

One can rearrange the terms of the first-order condition to be

\[
R(p) - 1 - r_l = \boxed{F + B - r_l} \left[ 1 - \frac{B}{F + B} \cdot \frac{1}{\alpha} \right].
\]
It shows that if the firm offers a compensation package that satisfies
\[ \alpha = \frac{B - r_l}{F + B - r_l} = \frac{B}{F + B}, \]
then the manager will choose the first-best \( p^0 \), which satisfies \( R(p^0) - 1 - r_l = 0 \).

Solving these two equations together, in equilibrium choices are
\[
\begin{align*}
B_e^* &= \frac{\bar{U}(I - A)}{p_e^* (p_e^* + r_l - \bar{U})} - \frac{\bar{U}r_l(1 - p_e^*)}{p_e^* (p_e^* + r_l)}, \\
F_e^* &= \frac{I - A - (1 - p_e^*)r_l}{p_e^*} + \frac{\bar{U}r_l(1 - p_e^*)}{p_e^* (p_e^* + r_l)}, \\
\alpha_e^* &= \frac{B_e^*}{F_e^* + B_e^*} = \frac{\bar{U}}{p_e^* + r_l},
\end{align*}
\]
where in equilibrium \( p_e^* = p^0 \). One can show that \( B_e^* \) is always positive because \( p_e^* (p_e^* + r_l - \bar{U}) < p_e^* (p_e^* + r_l) \) and \( (I - A) > r_l(1 - p_e^*) \).

For comparative statics, we first note that
\[ \frac{dp^0}{dr_l} = \frac{d \exp[r_f - 1 - r_l]}{dr_l} = -\exp[r_f - 1 - r_l] = -p^0. \]

The comparative statistics with respect to \( r_l \) can be calculated explicitly as
\[
\frac{d\alpha_e^*}{dr_l} = \frac{\frac{-\bar{U}}{(p^0 + r_l)^2} \frac{d(p^0 + r_l)}{dr_l}}{\frac{d\bar{U}}{dr_l}} = \frac{-\bar{U}}{(p^0 + r_l)^2} \left[1 - p^0\right] < 0
\]
And in the same way, we can see that
\[
\frac{dF_e^*}{dr_l} = \frac{d \left( \frac{I - A - r_l}{p^0} + r_l + \frac{p_e^* r_l (1 - p^0)}{p^0 (p^0 + r_l)} \right)}{dr_l} = \frac{I - A - r_l - 1 + p^0}{p^0} + \bar{U} \left[1 - p^0 + r_l (1 + p^0)\right] (p^0 + r_l) + 2p^0 r_l (1 - p^0) \frac{p^0 (p^0 + r_l)^2}{> 0}
\]
Thus one can obtain \( \frac{dF_e^*}{dr_l} > 0 \), when the debt borrowing \( I - A \) is sufficiently large. When \( I - A \)
is small, \( dF_e^*/dr_l \) could be negative when \( p^o \) is very low. Following the same method,

\[
\frac{dB_e^*}{dr_l} = \frac{d}{dr_l} \left( \frac{\overline{U}(I-A)}{p^o[p^o + r_l - \overline{U}]} - \frac{\overline{U}r_l(1-p^o)}{p^o(p^o + r_l)} \right)
\]

\[
= \frac{\overline{U}(I-A)(2p^o + r_l - 1)}{p^o(p^o + r_l - \overline{U})^2} - \frac{\overline{U}[1 - p^o + r_l(1 + p^o)](p^o + r_l) + 2p^o r_l \cdot (1 - p^o)}{p^o(p^o + r_l)^2},
\]

which has an ambiguous sign.

The comparative statistics with respect to \( \overline{U} \) can be calculated explicitly as

\[
\frac{dF_e^*}{d\overline{U}} = \frac{r_l \cdot (1 - p^o)}{p^o[p^o + r_l]} > 0,
\]

\[
\frac{d\alpha^*_e}{d\overline{U}} = \frac{1}{p^o + r_l} > 0.
\]

Note that the total derivative of \( \alpha^*_e \) with respect to \( \overline{U} \) is given by

\[
\frac{d\alpha^*_e}{d\overline{U}} = \frac{\partial \alpha^*_e}{dF_e^*} \frac{dF_e^*}{d\overline{U}} + \frac{\partial \alpha^*_e}{dB_e^*} \frac{dB_e^*}{d\overline{U}} > 0,
\]

which suggests that \( dB_e^*/d\overline{U} > 0 \).

**Proof of Proposition 2, Corollary 2, and part of Corollaries 4 and 5**

From the manager’s first-order condition to maximize his payoff, we see that

\[
\frac{\partial U_M}{\partial p} = \alpha [R(p) - F - B - 1] + B = 0 \quad (11)
\]

gives

\[
R(p) - 1 - r_l = F + B - r_l - \frac{B}{\alpha} \quad (12)
\]

That is, to induce the manager to choose \( p^o \), the firm offers a compensation package that satisfies \( \alpha = B/(F + B - r_l) \). Given the optimal risk level \( p^o \) and \( \alpha = B/(F + B - r_l) \), the firm’s problem
is to select $F$ and $B$ in order to satisfy

$$U_C = p^o \cdot F + (1 - p^o) \cdot r_l - (I - A) = 0,$$

$$U_M = \alpha \cdot p^o \cdot (R(p^o) - F - B) + p^o \cdot B = U.$$

Solve these two equations together, in equilibrium, the optimal performance-based compensation is

$$B^*_c = \frac{U(I - A - r_l)}{p^*_c (p^*_c - U)} > 0, F^*_c = \frac{I - A - (1 - p^*_c) r_l}{p^*_c}, \alpha^*_c = \frac{U}{p^*_c},$$

where in equilibrium $p^*_c = p^o$.

The comparative statistics with respect to $r_l$ can be calculated explicitly as

$$\frac{d\alpha^*_c}{dr_l} = \frac{d}{dr_l} \frac{U}{p^o} = \frac{-U}{(p^o)^2} \frac{d(p^o)}{dr_l} = \frac{U}{p^o} > 0$$

And in the same way, we can see that

$$\frac{dF^*_c}{dr_l} = \frac{I - A - r_l - 1 + p^o}{p^o}$$

Thus one can obtain $dF^*_c/dr_l > 0$, when the debt borrowing $I - A$ is sufficiently large. Otherwise, $dF^*_c/dr_l < 0$. It is also easy to verify that $\frac{dF^*_c}{dr_l} < \frac{dB^*_c}{dr_l}$. Also, we have

$$\frac{dB^*_c}{dr_l} = \frac{2p^o (I - A - r_l) - (p^o - U)}{p^o (p^o - U)^2},$$

which can be positive or negative.

The comparative statistics with respect to $U$ can be calculated explicitly as

$$\frac{dF^*_c}{dU} = 0, \frac{d\alpha^*_c}{dU} = \frac{1}{p^o} > 0.$$

Note that the total derivative of $\alpha^*_c$ with respective to $U$ is given by

$$\frac{d\alpha^*_c}{dU} = \frac{\partial \alpha^*_c}{\partial F^*_c} \frac{dF^*_c}{dU} + \frac{\partial \alpha^*_c}{\partial B^*_c} \frac{dB^*_c}{dU} > 0,$$
which suggests that
\[ \frac{dB_c^*}{dU} = \frac{(I - A - r_l)}{(p^0 - U)^2} > 0. \]

**Proof of Proposition 3**

The manager’s first-order condition to make the risk choice is:

\[ \frac{\partial U_M}{\partial p} = \alpha [R(p) - F - B - 1] + B - \min\{r_l, B\} = 0, \]

which can be rewritten to be

\[ R(p) = 1 + r_l + \left( F + B - r_l - \frac{B - \min\{r_l, B\}}{\alpha} \right). \]

As long as the firm offers a compensation package that satisfies

\[ \alpha = \frac{B - \min\{r_l, B\}}{F + B - r_l}, \]

the manager will choose \( p^0 \).

(i) The case of \( U > r_l \): we conjecture that the inside debt \( B \) is higher than \( r_l \) (i.e., \( \min\{r_l, B\} = r_l \)). The compensation must satisfy the following condition to induce first-best risk choice:

\[ \alpha = \frac{B - \min\{r_l, B\}}{F + B - r_l} = \frac{B - r_l}{F + B - r_l}. \]

The creditor’s payoff and the manager’s expected payoff are characterized by respectively

\[ U_C = p \cdot F - (I - A) = 0, \]
\[ U_M = \alpha \cdot p \cdot (R(p) - F - B) + p \cdot B + (1 - p) \cdot r_l = U. \]

Solving the equilibrium, we have

\[ \alpha^*_m = \frac{\bar{U} - r_l}{p^0}, B^*_m = r_l + \frac{(I - A)(\bar{U} - r_l)}{p^0 (p^0 + r_l - \bar{U})}, \]
\[ F^*_m = \frac{I - A}{p^0}, p^*_m = p^0. \]
Notice that the equilibrium inside debt $B^*_{m1}$ is indeed larger than $r_l$, which confirms our initial conjecture.

(ii) The case of $\bar{U} \leq r_l$: we conjecture that the inside debt $B$ is smaller than $r_l$, that is, $\min\{r_l, B\} = B$ or $r_l > B \geq \bar{U}$. In this case, we have:

$$\alpha = \frac{B - \min\{r_l, B\}}{F + B - r_l} = 0.$$ 

Suppose that $\alpha = 0$, then the manager’s expected payoff is

$$U_M = \alpha \cdot p \cdot (R(p) - F - B) + p \cdot B + (1 - p) \min\{r_l, B\} = B \geq \bar{U}.$$ 

This implies that $r_l > \min\{r_l, B\} = B \geq \bar{U}$. The manager will receive a fixed inside debt in any cases. As the manager is indifferent, we assume that the manager will implement the first-best project selection $p^o$. In this case, the creditor’s expected payoff is

$$U_C = p^o \cdot F + (1 - p^o) \cdot (r_l - B) - (I - A) = 0.$$ 

We can obtain the equilibrium debt repayment,

$$F = \frac{(I - A) - (1 - p) \cdot (r_l - B)}{p^o},$$

which makes sure that the creditor is break even. Thus, when $r_l > \min\{r_l, B\} = B \geq \bar{U}$, in equilibrium we have

$$\alpha^*_m = 0, B^*_m = \bar{U}, F^*_m = \frac{(I - A) - (1 - p^o) \cdot (r_l - \bar{U})}{p^o}, p^*_m = p^o.$$

Proof of Corollary 3, and part of Corollaries 4 and 5

The proof of Corollary 3 is derived in the text. For the comparative statics analysis, when
\( \bar{U} > r_l \), following a similar analysis, by applying Cramer’s rule we obtain

\[
\frac{dF_{m1}^*}{dr_l} = \frac{d}{dr_l} \left( I - A \right) \frac{I - A}{p^o} > 0,
\]

\[
\frac{d\alpha_{m1}^*}{dr_l} = \frac{d}{dr_l} \left( \frac{\bar{U} - r_l}{p^o} \right) \frac{\bar{U} - r_l - 1}{p^o} < 0,
\]

and

\[
\frac{dB_{m1}^*}{dr_l} = \frac{d}{dr_l} \left( r_l + \frac{(I - A)(\bar{U} - r_l)}{p^o(p^o + r_l - \bar{U})} \right) = 1 + (I - A) \frac{(2p^o + r_l)(\bar{U} - r_l) - p^o}{p^o(p^o + r_l - \bar{U})^2} > 0;
\]

So \( dB_{m1}^*/dr_l \) can be positive or negative.

We can calculate the comparative statics as follows:

\[
\frac{dB_{m1}^*}{dU} = \frac{1}{p^o} \frac{(I - A)(p^o + r_l - \bar{U}) + (I - A)(\bar{U} - r_l)}{(p^o + r_l - \bar{U})^2} > 0;
\]

\[
\frac{dF_{m1}^*}{dU} = 0, \frac{d\alpha_{m1}^*}{dU} = \frac{1}{p^o} > 0.
\]

In the same way, we show that when \( U < r_l \), \( \frac{d\alpha_{m2}^*}{dr_l} = \frac{dB_{m2}^*}{dr_l} = 0 \), but \( \frac{dF_{m2}^*}{dr_l} > 0 \). In addition, \( \frac{d\alpha_{m2}^*}{dp^o} = 0 \), but \( \frac{dB_{m2}^*}{dp^o} \) and \( \frac{dF_{m2}^*}{dp^o} \) are both positive.

**Proof of Propositions 4 and 5**

The results in Proposition 5 are quite straightforward. It is obvious that \( \alpha_{e}^* = \frac{\bar{U}}{p^o} > \alpha_{e}^* = \frac{\bar{U}}{p^o + r_l} \)

and \( \alpha_{m1}^* = \frac{\bar{U} - r_l}{p^o} > \alpha_{m2}^* = 0 \). To show \( \alpha_{e}^* > \alpha_{m1}^* \), we have

\[
\alpha_{e}^* - \alpha_{m1}^* = \frac{\bar{U}}{p^o + r_l} - \frac{\bar{U} - r_l}{p^o} = \frac{\bar{U}p^o - (\bar{U} - r_l)(p^o + r_l)}{(p^o + r_l)p^o} = \frac{(p^o + r_l - \bar{U})r_l}{(p^o + r_l)p^o} > 0.
\]
Similarly, it is obvious that $F_c^* = \frac{I - A - (1-p^o)r_l}{p^o} < F_e^* = \frac{I - A - (1-p^p)r_l}{p^p} + \frac{U r_l (1-p^p)}{p^p[p^p + r_l]}$ and $F_{m2}^* = \frac{(I-A)-(1-p^o)(r_l-U)}{p^o} < F_{m1}^* = \frac{I - A}{p^p}$. To show $F_e^* < F_{m2}^*$, we have

$$F_e^* - F_{m2}^* = \frac{\bar{U} \cdot r_l \cdot (1-p^o)}{p^p[p^o + r_l]} - \frac{(1-p^o) \cdot \bar{U}}{p^o} = -\frac{(1-p^o) \cdot \bar{U}}{p^o + r_l} < 0.$$ 

For Proposition 4, when $\bar{U} > r_l$, we identify the following properties:

<table>
<thead>
<tr>
<th>Case</th>
<th>Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_{m1}^* = B_{e}^*$</td>
<td>$I - A = \frac{(p^o - \bar{U} + r_l)(p^o^2 + \bar{U} - p^o \bar{U} + p^o r_l)}{p^o + r_l}$;</td>
</tr>
<tr>
<td>$B_{e}^* = B_{e}^*$</td>
<td>$I - A = \frac{(p^o - \bar{U} + r_l)(p^o^2 + \bar{U} - p^o \bar{U} + p^o r_l)}{p^o + r_l}$;</td>
</tr>
<tr>
<td>$B_{M1}^* = B_{e}^*$</td>
<td>$I - A = \frac{(p^o - \bar{U} + r_l)(p^o^2 + \bar{U} - p^o \bar{U})}{p^o}$;</td>
</tr>
</tbody>
</table>

where by assumption $p^o + r_l - \bar{U} > I > 0$. Then we can show

<table>
<thead>
<tr>
<th>Case</th>
<th>Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_{m1}^* &gt; B_{e}^*$</td>
<td>$I - A &lt; \frac{(p^o - \bar{U} + r_l)(p^o^2 + \bar{U} - p^o \bar{U} + p^o r_l)}{p^o + r_l}$;</td>
</tr>
<tr>
<td>$B_{e}^* &gt; B_{e}^*$</td>
<td>$I - A &lt; \frac{(p^o - \bar{U} + r_l)(p^o^2 + \bar{U} - p^o \bar{U} + p^o r_l)}{p^o + r_l}$;</td>
</tr>
<tr>
<td>$B_{M1}^* &gt; B_{e}^*$</td>
<td>$I - A &lt; \frac{(p^o - \bar{U} + r_l)(p^o^2 + \bar{U} - p^o \bar{U})}{p^o}$;</td>
</tr>
</tbody>
</table>

The thresholds are ordered by

$$\frac{(p^o - \bar{U} + r_l)(p^o^2 + \bar{U} - p^o \bar{U} + r_l)}{p^o + r_l} > \frac{(p^o - \bar{U} + r_l)(p^o^2 + \bar{U} - p^o \bar{U})}{p^o} > \frac{(p^o - \bar{U} + r_l)(p^o^2 + \bar{U} - p^o \bar{U} + p^o r_l)}{p^o + r_l}.$$
Taken together, we can show the following cases:

<table>
<thead>
<tr>
<th>Case</th>
<th>Case Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_{M1}^* &gt; B_e^* &gt; B_c^*$</td>
<td>I-1 $I - A &lt; \frac{[p^o + \bar{U}(1-p^o) + p^o r_l][p^o - \bar{U} + r_l]}{p^o + r_l}$;</td>
</tr>
<tr>
<td>$B_e^* &gt; B_{M1}^* &gt; B_c^*$</td>
<td>I-2 $\frac{[p^o + \bar{U}(1-p^o) + p^o r_l][p^o - \bar{U} + r_l]}{p^o + r_l} &lt; I - A &lt; \frac{[p^o + \bar{U}(1-p^o)][p^o - \bar{U} + r_l]}{p^o + r_l}$;</td>
</tr>
<tr>
<td>$B_e^* &gt; B_c^* &gt; B_{M1}^*$</td>
<td>I-3 $\frac{[p^o + \bar{U}(1-p^o)][p^o - \bar{U} + r_l]}{p^o + r_l} &lt; I - A &lt; \frac{(p^o + \bar{U}(1-p^o) + r_l)(p^o - \bar{U} + r_l)}{p^o + r_l}$;</td>
</tr>
<tr>
<td>$B_c^* &gt; B_e^* &gt; B_{M1}^*$</td>
<td>I-4 $I - A &gt; \frac{(p^o + \bar{U}(1-p^o) + r_l)(p^o - \bar{U} + r_l)}{p^o + r_l}$.</td>
</tr>
</tbody>
</table>

In the same way, when $r_l > \bar{U}$, we can show the following cases:

<table>
<thead>
<tr>
<th>Case</th>
<th>Case Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_{M2}^* &gt; B_e^* &gt; B_c^*$</td>
<td>II-1 $I - A &lt; \frac{(p^o + r_l)(p^o - \bar{U} + r_l)}{p^o + r_l}$;</td>
</tr>
<tr>
<td>$B_e^* &gt; B_{M2}^* &gt; B_c^*$</td>
<td>II-2 $\frac{(p^o + r_l)(p^o - \bar{U} + r_l)}{p^o + r_l} &lt; I - A &lt; p^o(p^o - \bar{U}) + r_l$;</td>
</tr>
<tr>
<td>$B_e^* &gt; B_c^* &gt; B_{M2}^*$</td>
<td>II-3 $p^o(p^o - \bar{U}) + r_l &lt; I - A &lt; \frac{(p^o + \bar{U}(1-p^o) + r_l)(p^o - \bar{U} + r_l)}{p^o + r_l}$;</td>
</tr>
<tr>
<td>$B_c^* &gt; B_e^* &gt; B_{M2}^*$</td>
<td>II-4 $I - A &gt; \frac{(p^o + \bar{U}(1-p^o) + r_l)(p^o - \bar{U} + r_l)}{p^o + r_l}$.</td>
</tr>
</tbody>
</table>